The Tensor Presentation of Infocommunications Networks on the Basis of Toroidal Structures

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Abstract - The toroidal topology structures and examples of infocommunications networks construction were considered. The tensor presentation of topology structure for toroidal networks is proposed. The compound-tensor is entered and the decomposition of difficult topology structure was conducted.

Keywords - toroidal topology, tensor presentation, compound-tensor.

INTRODUCTION

Subsequent development of the telecommunication systems is closely related to the construction of next generation network - NGN. The task of designer consists in the search of optimum ways of transition from today network to the NGN. Data transferring in the NGN is a packet, and from practical using properties of self-similarity or scale invariance of statistical descriptions found out for this method. In consideration of revealed network processes features the questions of fractal researching methods development of modern telecommunication networks and estimation of its influence for the specifics of packet data transmission are getting a special actuality. In this case the system of virtual connections state prognostication have to become a key part in the structure of the distributed network management of processes, in which the features of network traffic stochastic nature are taken into account. In these terms development of new network technologies and increasing of modern telecommunications systems performance efficiency require a creation of mathematical models which will fully represent the properties of processes marked higher in networks.

TOROIDAL TOPOLOGY STRUCTURES FOR INFOMCOMUNICATIONS NETWORKS

Generalized chart of transport network with toroidal structure are shown in Fig.1 [2]. It consists of m basic structures exemplars L₁, in which tips with equal number connected with each other in order to the structure L₂. That structure can be constructed with using of complicated structures formation algorithm for two-dimensional case (k = 2).

1. Formation of structure L₁ n exemplars. Two-digit identification number Aᵢ(p,τ) assign to each from N=mn elements of structure L, where p – is the number of exemplar (basic structure), τ – low bit – number of the element in the exemplar.

2. Association of the elements with equal numbers of low digits for all the exemplars of structure L according to its incidence relation. In the capacity of L1 and L2 circles with some of tips are chosen and L=L₁×L₂ is formed. As a result the toroidal structure is received which consists of m circles each contains n tips, coupled by interconnection cycles with each other.

Sets, which are disjoint in the cycles nodes, length and number of which depend from chosen interconnection cycle step, represent the network of interconnections Lᵣ, which cover the nodes hookup of structure L. The sequence of uniform networks with constant number of data links can be getting by changing of cycle step, in which the mean length and network diameter are varying over a wide range. Thereby, there is an opportunity to form digital network with a flexible topology. It is important to say there is a possibility to correct the network structural characteristics over a wide range and significantly to change the network topology in consideration of network loading based on the toroidal structure. In general, practical utilization shows that in the case of toroidal networks the nodes of every basic structure are connected with the next nodes by numeration in opposite to situation shown on Fig.1 (here L₁ and L₂ are equal). This is equivalent to the cyclic increment shift of numeration. In this case the number difference between incidence to some interconnection link (the links of structure Lᵣ) nodes is the magnitude equal to St=n+k named the interconnection cycle step, where k = 1, 2, ... is an appropriate shift. In effort [5] two-part uniform network with incidence relations set as some ridge shuffling is defined. Similarly, the distribution of tips for sets, which conforms to different interconnection cycles and sets the topology elements connection rule, defines the value of the step in toroidal networks. For some non-negative integers St, which are a set of N=mn tips, those St are grouping into two sets of cycles:

\[ V_i = v_i(p) + \tau, \quad V_c = v_c(p,c) + St \tau_c \mod N \]  \hspace{1cm} (1)
where $V_1 (V_c)$ is a set of tips in structure $L_1 (L_c)$ cycles; $p=1, \ldots, m$; $\tau=0, \ldots, n-1$; $p_c=1, \ldots, w$; $\tau_c=0, \ldots, A-1$; $w$ is a quantity of cycles with the length $A$ in the interconnection network; $v_1(p)$ is the number of initial tip in the $p$-$m$ cycle of structure $L_1$; $v_c(p_c)$ is the number of initial tip in the $p_c$-$m$ cycle of structure $L_c$. And also:

$$v_1(p)=1+n(p-1), \quad v_c(p_c)=w+p_c-1. \quad (2)$$

The examples of toroidal uniform network construction, when $m=n=6$, the quantity of ridges is $M=2nm=72$ for cases, when the steps are $St=1$, $St=8$, $St=10$, $St=15$, are shown at Fig.2, a – d [2].

Tensor $T_{\alpha\beta\gamma\delta\epsilon\zeta}$, which describe the connection between tips in toroidal networks, is proposed. The element of tensor equals to 1 in the case, when there is a ridge between tips and in opposite case it equals to 0. A loopback exists, when $T_{\alpha\beta\gamma\delta\epsilon\zeta} \delta_{\alpha\gamma} \delta_{\beta\delta}$ equals to 1. This loopback symbolize entrance and exit of the same tip and explains the packages processing in the tip. Examined tensor is a symmetric according to a pair of indexes. It is combined (1) and (2) for transition to matrix form $T_{\alpha\beta}$, and number of each tip is defined. This expressions present at such form as:

$$v=n(\alpha-1)+\beta \quad (3)$$

Exactly tensor can be presented easier as compound-tensor $K_{\alpha\beta}$[6]. Diagonal tensors $K_{\alpha\beta} \delta_{\alpha\beta} = T_{\alpha\beta\gamma\delta\epsilon\zeta} \delta_{\alpha\gamma} \delta_{\beta\delta} = c_{\beta\beta}$, are equal and describe the connection between the elements in the exemplars. For all cases are shown at Fig.2:

$$c_{\beta\beta}=\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 1
\end{bmatrix} \quad (4)$$

Tensor $T_{\alpha\beta}$ has shifted at $k$ amount ones matrix except the diagonal tensors $c_{\beta\beta}$. Thus, it is also two compound-tensors in general $T_{\alpha\beta\gamma\delta\epsilon\zeta} \delta_{\alpha\gamma} \delta_{\beta\delta} = I_1$ and $T_{\alpha\beta\gamma\delta\epsilon\zeta} \delta_{\alpha\gamma} \delta_{\beta\delta} = I_2$. In the case shown at Fig.2b using compound tensors, it is written as:

$$T_{\alpha\beta}=\begin{bmatrix}
\epsilon_{\beta\beta} & I_1 & I_2 & 0 & I_2 & I_1 \\
I_1 & \epsilon_{\beta\beta} & I_2 & 0 & I_2 & 0 \\
I_2 & I_1 & \epsilon_{\beta\beta} & 0 & I_2 & 0 \\
0 & I_2 & I_1 & \epsilon_{\beta\beta} & I_2 & 0 \\
I_2 & 0 & I_2 & I_1 & \epsilon_{\beta\beta} & 0 \\
0 & I_1 & 0 & I_2 & I_1 & \epsilon_{\beta\beta}
\end{bmatrix} \quad (5)$$

0 – is a tensor which consists of zero components.

There is an opportunity to define a quantity of ridges between all the exemplars of basic structures by summing up through second and fourth indexes:

$$b_{\alpha\beta}=\sum_{\beta,\epsilon} T_{\alpha\beta\epsilon\zeta} \delta_{\alpha\gamma} \delta_{\beta\delta} \delta_{\epsilon\gamma} \delta_{\zeta\delta} \quad \beta,\epsilon \neq \alpha, \delta \quad \quad (6)$$

Formula (6) is of the form (7) for the case shown at Fig.2b:

$$b_{\alpha\beta}=\begin{bmatrix}
18 & 4 & 2 & 0 & 2 & 4 \\
4 & 18 & 4 & 2 & 0 & 2 \\
2 & 4 & 18 & 4 & 2 & 0 \\
0 & 2 & 4 & 18 & 4 & 2 \\
2 & 0 & 2 & 4 & 18 & 4 \\
4 & 2 & 0 & 2 & 4 & 18
\end{bmatrix} \quad (7)$$

At (7) diagonal elements $b_{\alpha\beta} \delta_{\alpha\beta}$ equals 18, although at Fig.2 in every exemplar of basic structure there is 5 ridges, but each tip has two connections with other taken into account one loopback connections. Total amount of tips is 6, therefore $6 \times (2+1)=18$.

CONCLUSION

Tensor presentation of toroidal topological structure, which allows performing an investigation and synthesis of infocommunication networks, is proposed. Tensor analysis of toroidal topology structures is conducted. Compound-tensor for decomposition of complicated topological structure is introduced.

REFERENCES