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SOFTWARE-ALGORITHMIC SUPPORT OF FINITE-ELEMENT ANALYSIS OF SPATIAL THERMOVALENTRANSLATIONS IN ANISOTROPIC CAPILLARY-POROUS MATERIALS

On the basis of a three-dimensional mathematical model of nonisothermal moisture transfer in capillary-porous materials, taking into account the anisotropy of thermophysical properties, a software complex was developed for conducting finite-element analysis of bound thermal gravity with the use of CUDA technology.

Key words: software, mathematical model, heat transfer, finite elements method.

ПРОГРАМНО-АЛГОРИТМІЧНЕ ЗАБЕЗПЕЧЕННЯ СКІНЧЕННОЕЛЕМЕНТНОГО АНАЛІЗУ ПРОСТОРОВОГО ТЕПЛОВОЛОГОПЕРЕНЕСЕННЯ В АНІЗОТРОПНИХ КАПІЛЯРНО-ПОРИСТИХ МАТЕРІАЛАХ

На основі тривимірної математичної моделі неізотермічного вологоперенесення у капілярно-пористих матеріалах з урахуванням анізотропії теплофізичних властивостей розроблено програмний комплекс для виконання скінченноелементного аналізу зв'язаного тепловологоперенесення з використанням технології CUDA.

Ключові слова: програмне забезпечення, математична модель, тепловологоперенесення, метод скінченних елементів.

Introduction

Existing technologies of drying wood require constant improvement by conducting researches of non-stationary interrelated temperature-humid fields in dried capillary-porous materials. Both one-dimensional and two-dimensional linear and nonlinear mathematical models of heat transfer through the anisotropy of thermophysical properties are constructed today, but the study of the spatial case of nonisothermal moisture transfer remains relevant. Such a task is quite difficult for analytical solving. The application of approximate methods does not reflect an adequate picture of the interconnected fields of heat and moisture transfer. Therefore, for the analysis of the process of drying of capillary-porous materials, the construction of a three-dimensional mathematical model and the choice of an effective numerical method for its implementation are relevant. Therefore, the actual task is to create an object-oriented software complex, which would include the implementation of all stages of the method of finite elements. Such software packages should have a user-friendly user interface with rich graphical visualization capabilities for the finite-element splitting of the results of calculations, efficient methods of storing and importing data.

Analysis of existing research

The mathematical model of the thermodynamic transfer is described by differential equations in partial derivatives with boundary conditions of the third kind. In previous studies, a two-dimensional linear mathematical model of moisture transfer has been constructed taking into account the anisotropy of the thermophysical characteristics of the material [1, 2] and a two-dimensional nonlinear mathematical model of nonisothermal moisture transfer [3, 4]. Such mathematical modules are so complex that the use of analytic methods, such as the Fourier transform, Laplace, or in the form of a power series, is impossible or not practical, therefore, we have to resort to the search for the numerical approximations of an unknown analytic solution. In recent decades, numerous methods for solving non-stationary field field problems are rapidly developing, which are described by differential equations in partial derivatives, in particular, various methods [7]. Their essence consists in replacing the derivatives with finite-difference relations and in the exact execution of algebraic equations in the nodal points of the grid. However, during the application of these methods, finite-difference schemes of low order are often insufficiently accurate on coarse grids. At the level with the difference methods of widespread use, the method of finite elements was obtained, which involves the approximation of a continuous value by a discrete model on a finite number of sub-regions. The finite element method in implementation is more complicated than the finite difference method, but it has a number of advantages that are quite significant in real problems, namely: the ability to set the studied region in an arbitrary form; the construction of a thicker grid in places with a large gradient of investigated size, but less dense, where special precision is not required. In detailed mathematical studies, it turned out that for non-smooth input data of a problem, the finite element method often converges faster than the finite difference method, and sometimes it has an optimal rate of convergence [12–13]. FEM was proposed in the underlying work, but at that time, the relevance of this work was not recognized because of the lack of the possibility of implementing the method with the help of computers, and the progressive idea was forgotten. Subsequently, the ideas of the method of finite elements were developed in the works [7–8].

For a long time, the widespread distribution of the finite element method was hindered by the lack of algorithms for automatically splitting the studied region into finite elements. This problem has been solved with the advent of algorithms that sample the domain based on the Dellone criterion [11]. An efficient and reliable algorithm for constructing the Delaunay triangulation was described in [10–12].

One of the problems is also the creation of a fast software package, which would include the implementation of all stages of the finite element method. Since implementation of domain discretization is a process that uses a large amount of processor resource actuality, it is gaining momentum and accelerating the program that is implemented using parallel technologies, namely CUDA technology [17–22].

Therefore, for the analysis of the process of drying of capillary-porous materials, it is actual construction of a three-dimensional mathematical model and the creation of an effective software complex, which would include the use of parallel computing technology CUDA.

Mathematical model

The three-dimensional mathematical model of the interconnected heat and mass transfer in the process of drying capillary-porous materials, taking into account the anisotropy of thermophysical properties, is described by a system of differential equations in partial derivatives:

$$\rho_0 c \frac{\partial T(x,y,z,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_x \frac{\partial T(x,y,z,\tau)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda_y \frac{\partial T(x,y,z,\tau)}{\partial y}\right) + \quad (1)$$

$$+\frac{\partial U(x,y,z,\tau)}{\partial \tau} = \frac{\partial}{\partial x}\left(a_{m_x} \frac{\partial U(x,y,z,\tau)}{\partial x}\right) + \frac{\partial}{\partial y}\left(a_{m_y} \frac{\partial U(x,y,z,\tau)}{\partial y}\right) + \frac{\partial}{\partial z}\left(a_{m_z} \frac{\partial U(x,y,z,\tau)}{\partial z}\right) \quad (2)$$

with boundary conditions:

$$\lambda_n \frac{\partial T}{\partial n}\Big|_{x=S} + \rho_0(1-\varepsilon) r \beta_n \left(U|_{x=S} - U_{p_n}\right) = \alpha_n (T_c - T|_{x=S});$$
$$\left(a_{m_n}\frac{\partial U}{\partial n} + a_{m_n}\sigma \frac{\partial T}{\partial n}\right)\Big|_{x=S} = \beta_n\left(U_{p_n} - U|_{x=S}\right); \quad (3)$$
$$\left(a_{m_n}\frac{\partial U}{\partial n} + a_{m_n}\sigma \frac{\partial T}{\partial n}\right)\Big|_{x=0} = 0; \qquad \frac{\partial T}{\partial n}\Big|_{x=0} = 0.$$

and the initial conditions for an irregular moisture removal regime:

$$T|_{\tau=0} = T_0(x,y,z); \quad U|_{\tau=0} = U_0(x,y,z). \quad (4)$$

where $T(x, y, z, \tau)$, $U(x, y, z, \tau)$, – desired functions in the area

$$V = \{(x, y, z): x \epsilon[0, R_1], y \epsilon[0, R_2], z \epsilon[0, R_3], \tau \epsilon[0, \tau^*]\};$$

FEM – finite element method, $T(x, y, z, \tau)$ – desired temperature function; $U(x, y, z, \tau)$ – function desired moisture content; R_1, R_2, R_3 – geometric dimensions of lumber; S – geometric surface of the region; τ^* – time of the process; c – heat capacity of wood; ρ_0– heat capacity and base density of wood; $\lambda_1, \lambda_2, \lambda_3$– coefficients of thermal conductivity in directions of anisotropy; ε – phase transition criterion; τ – current time; r – specific heat of vaporization; α_n – heat transfer coefficient; a_{mx}, a_{my}, a_{mz} – coefficients of conductivity in directions of anisotropy; T_c – ambient temperature; U_p – equilibrium humidity; β_n – coefficient of water exchange; σ – thermogradier coefficient; n – vector normal to the surface.

For the regular regime, which is characterized by the corresponding values of the Fourier criterion numbers (F_0) and Bio (B_i), the initial conditions are given by spatial quadratic functions. To develop software for numerical realization of finite-element analysis of thermovalentranslations in capillary-porous materials.

Algorithmic aspects

The numerical implementation of the mathematical model (1)–(4) uses the finite element method (FEM), according to which it is necessary to obtain an equivalent variational formulation of the mathematical model of heat and mass transfer. It was assumed that the change of volunteer can be given as a sum of quantities due to gradients of moisture content and temperature $\frac{\partial U}{\partial \tau} = \frac{\partial U_1}{\partial \tau} + \frac{\partial U_2}{\partial \tau}$. Then equation (2) can be written:

$$\frac{\partial U_1}{\partial \tau} = \frac{\partial}{\partial x}\left(a_{m_x}\frac{\partial U_1}{\partial x}\right) + \frac{\partial}{\partial y}\left(a_{m_y}\frac{\partial U_1}{\partial y}\right) + \frac{\partial}{\partial z}\left(a_{m_z}\frac{\partial U_1}{\partial z}\right); \tag{5}$$

$$\frac{\partial U_2}{\partial \tau} = \frac{\partial}{\partial x}\left(a_{m_x}\sigma\frac{\partial U_2}{\partial x}\right) + \frac{\partial}{\partial y}\left(a_{m_y}\sigma\frac{\partial U_2}{\partial y}\right) + \frac{\partial}{\partial z}\left(a_{m_z}\sigma\frac{\partial U_2}{\partial z}\right). \tag{6}$$

The realization of a mathematical model in accordance with the concept of the finite element method [1–4, 20], taking into account the above assumption, is reduced to the solution of an equivalent variational problem on the basis of minimization of such functionalities:

$$I = \int_V \frac{1}{2}\left(\lambda_x\left(\frac{\partial T}{\partial x}\right)^2 + \lambda_y\left(\frac{\partial T}{\partial y}\right)^2 + \lambda_z\left(\frac{\partial T}{\partial z}\right)^2 + 2c\rho_0\frac{\partial T}{\partial \tau}T - 2\rho_0 r\varepsilon\frac{\partial U}{\partial \tau}T\right)dV + \int_S \left(\frac{\alpha}{2}(T_c - T)^2 - \rho_0 r(1-\varepsilon)\beta(U - U_p)T\right)dS; \tag{7}$$

$$J_1 = \int_V \frac{1}{2}\left(a_{m_x}\rho_0\left(\frac{\partial U_1}{\partial x}\right)^2 + a_{m_y}\rho_0\left(\frac{\partial U_1}{\partial y}\right)^2 + a_{m_z}\rho_0\left(\frac{\partial U_1}{\partial z}\right)^2 + 2U\rho_0\frac{\partial U_1}{\partial \tau}\right)dV + \int_S \frac{1}{2}\rho_0\beta\left(U_1 - U_p\right)^2 dS; \tag{8}$$

$$J_2 = \int_V \frac{1}{2}\left(a_{m_x}\rho_0\sigma\left(\frac{\partial T}{\partial x}\right)^2 + a_{m_y}\rho_0\sigma\left(\frac{\partial T}{\partial y}\right)^2 + a_{m_z}\rho_0\sigma\left(\frac{\partial T}{\partial z}\right)^2 + 2T\rho_0\frac{\partial U_2}{\partial \tau}\right)dV. \tag{9}$$

For the approximation of the continuous value of the temperature T and the moisture content in the wood U on a discrete model, the investigated area V s divided into a finite number of elements. Then the moisture content $\{U^{(e)}\}$ and the temperature $\{T^{(e)}\}$ are determined for each sampling element:

$$\{T^{(e)}\} = [N^{(e)}]\{T\}, \{U^{(e)}\} = [N^{(e)}]\{U\},$$

introduce the notation

$$\{g_T^{(e)}\}^T = \left\{\frac{\partial T}{\partial x}\ \frac{\partial T}{\partial y}\ \frac{\partial T}{\partial z}\right\}, \qquad \{g_U^{(e)}\}^U = \left\{\frac{\partial U}{\partial x}\ \frac{\partial U}{\partial y}\ \frac{\partial U}{\partial z}\right\}.$$

then

$$\left\{g_T^{(e)}\right\}^T = \left[B^{(e)}\right]\{T\}, \qquad \left\{g_U^{(e)}\right\}^U = \left[B^{(e)}\right]\{U\},$$

where $\{U\}$ – vector of moisture content; $\{T\}$ – temperature vector; k – number of nodes; $\left[N^{(e)}\right]$ – function of form element; $\left[B^{(e)}\right]$ – gradient matrix.

As a finite element, the first order tetrahedron was chosen. The functions of the form of such an element have the form:

$$N_i = \frac{1}{6V}(a_i - b_i x - c_i y - d_i z). \tag{10}$$

It is necessary to consider the contribution of a separate element of the partition in the functional $, J_1, and\ J_2$, then (7)-(9) can be written as follows:

$$I = \sum_{e=1}^{n} \int_{V^{(n)}} \frac{1}{2}\Bigg(\{T^{(e)}\}\left[B^{(e)}\right]^T\left[Q^{(e)}\right]\left[B^{(e)}\right]\{T^{(e)}\}^T + 2c\rho_0\left[N^{(e)}\right]^T\left[N^{(e)}\right]\{T^{(e)}\}\frac{\partial\{T^{(e)}\}}{\partial\tau}$$

$$-\ 2\varepsilon\rho_0 r\left[N^{(e)}\right]^T\{T^{(e)}\}\frac{\partial\{U^{(e)}\}}{\partial\tau}\Bigg)dV^{(e)}$$

$$+\int_{S^{(e)}} \frac{1}{2}\Big(\alpha\left[N^{(e)}\right]^T\left[N^{(e)}\right]\{T^{(e)}\} - \alpha T_c\left[N^{(e)}\right]^T\{T^{(e)}\} + \alpha T_C^2$$

$$2\rho_0 r(1-\varepsilon)\beta\left(U - U_p\right)\left[N^{(e)}\right]^T\{T^{(e)}\}\Big)\,dS^{(e)}; \tag{11}$$

$$J_1 = \sum_{e=1}^{n} \int_{V^{(n)}} \frac{1}{2}\left(\{U^{(e)}\}\left[B^{(e)}\right]^T\left[D^{(e)}\right]\left[B^{(e)}\right]\{U^{(e)}\}^T + 2\rho_0\left[N^{(e)}\right]^T\left[N^{(e)}\right]\{U^{(e)}\}\frac{\partial\left\{U_1^{(e)}\right\}}{\partial\tau}\right)dV^{(e)}$$

$$+\int_{S^{(e)}} \frac{1}{2}\left(\rho_0\beta\left(\{U^{(e)}\}^T\left[N^{(e)}\right]^T\left[N^{(e)}\right]\{U^{(e)}\} - 2U_p\left[N^{(e)}\right]^T\{U^{(e)}\} + U_p^2\right)\right)dS^{(e)}; \tag{12}$$

$$J_2 = \sum_{e=1}^{n} \int_{V^{(n)}} \frac{1}{2}\left(\{T^{(e)}\}\left[B^{(e)}\right]^T\left[P^{(e)}\right]\left[B^{(e)}\right]\{T^{(e)}\}^T + 2\rho_0\left[N^{(e)}\right]^T\left[N^{(e)}\right]\{T^{(e)}\}\frac{\partial\left\{U_2^{(e)}\right\}}{\partial\tau}\right)dV^{(e)}; \tag{13}$$

where n – is the number of sampling elements of area V.

The minimum of functionals is achieved by differentiating the functionals (11)–(13) for $\{U\}$ and $\{T\}$, which results in:

$$\frac{\partial I}{\partial\{T\}} = \sum_{e=1}^{n} \int_{V^{(n)}} [B^{(e)}]^T\left[Q^{(e)}\right]\left[B^{(e)}\right]dV^{(e)}\{T^{(e)}\} + \int_{V^{(n)}} c\rho_0\left[N^{(e)}\right]^T[N^{(e)}]dV^{(e)}\frac{\partial\{T^{(e)}\}}{\partial\tau}$$

$$-\int_{V^{(e)}} \left(\varepsilon\rho_0 r\left[N^{(e)}\right]^T\right)dV\frac{\partial\{U^{(e)}\}}{\partial\tau} + \int_{S^{(e)}} \left(\alpha\left[N^{(e)}\right]^T\left[N^{(e)}\right]\right)dS^{(n)}\{T^{(e)}\} -$$

$$-\int_{S^{(e)}} \alpha T_c\left[N^{(e)}\right]^T dS^{(e)} - \int_{S^{(e)}} \rho_0 r(1-\varepsilon)\beta\left(U - U_p\right)\left[N^{(e)}\right]^T dS^{(n)}; \tag{14}$$

$$\frac{\partial J_1}{\partial\{U\}} = \sum_{e=1}^{n} \int_{V^{(n)}} [B^{(e)}]^T\left[D^{(e)}\right]\left[B^{(e)}\right]dV^{(e)}\{U^{(e)}\} + \int_{V^{(n)}} \rho_0\left[N^{(e)}\right]^T[N^{(e)}]dV^{(e)}\frac{\partial\{U_1{}^{(e)}\}}{\partial\tau} +$$

$$+\int_{V^{(e)}} \left(\rho_0\beta\left[N^{(e)}\right]^T[N^{(e)}]\right)dS\{U^{(e)}\} - \int_{S^{(e)}} \left(\rho_0\beta U_p\left[N^{(e)}\right]^T\right)dS^{(n)}; \tag{15}$$

$$\frac{\partial J_2}{\partial\{U\}} = \sum_{e=1}^{n} \int_{V^{(n)}} [B^{(e)}]^T [P^{(e)}][B^{(e)}] dV^{(e)} \{T^{(e)}\} \quad + \int_{V^{(n)}} \rho_0 [N^{(e)}]^T [N^{(e)}] dV^{(e)} \frac{\partial\{U_2{}^{(e)}\}}{\partial\tau}. \qquad (16)$$

Having grouped integrals, we introduce the following matrices:

$$[R] = \sum_{e=1}^{n} \int_{V^{(e)}} \left(c\rho_0 [N^{(e)}]^T [N^{(e)}]\right) dV^{(e)};$$

$$[K_1] = \sum_{e=1}^{n} \int_{V^{(e)}} \left([B^{(e)}]^T [Q^{(e)}][B^{(e)}]\right) dV^{(e)} + \int_{V^{(e)}} \left(\alpha [N^{(e)}]^T [N^{(e)}]\right) dS^{(e)};$$

$$[M''] = \sum_{e=1}^{n} \int_{V^{(e)}} \left(\varepsilon\rho_0 r [N^{(e)}]^T\right) dV^{(e)} \frac{\partial\{U^{(e)}\}}{\partial\tau} - \int_{S^{(e)}} \left(\alpha T_c [N^{(e)}]^T\right) dS^{(e)} -$$

$$- \int_{S^{(e)}} \left(\rho_0 r(1-\varepsilon)\beta(U-U_p)[N^{(e)}]^T\right) dS^{(e)};$$

$$[M] = \sum_{e=1}^{n} \int_{V^{(e)}} \left(\rho_0 [N^{(e)}]^T [N^{(e)}]\right) dV^{(e)};$$

$$[K] = \sum_{e=1}^{n} \int_{V^{(e)}} \left([B^{(e)}]^T [D^{(e)}][B^{(e)}]\right) dV^{(e)} + \int_{V^{(e)}} \left(\rho_0\beta [N^{(e)}]^T [N^{(e)}]\right) dS^{(e)};$$

$$[M'] = \sum_{e=1}^{n} \int_{V^{(e)}} \left([B^{(e)}]^T [P^{(e)}][B^{(e)}]\right) dV^{(e)} + \int_{V^{(e)}} \left(\rho_0\beta U_p [N^{(e)}]^T\right) dS^{(e)}.$$

Equating the relation (14)–(16) to zero and grouping integrals into it, we obtain the final system of ITU matrix equations for the realization of the mathematical model (1)–(4):

$$[R]\frac{\partial\{T\}}{\partial\tau} + [K_1]\{T\} + \{M''\} = 0, [M]\frac{\partial\{U\}}{\partial\tau} + [K]\{U\} + \{M'\} = 0\,. \qquad (17)$$

Finding the spatial functions of moisture content and temperature at any point in the time interval is reduced to finding a solution to the Cauchy problem for the system of differential equations (17) For the solution of the system, the algorithm of the "predictor-proofreader" type [9] is used.

An important stage in the finite-element analysis is the discretization of the investigated area, by constructing a three-dimensional finite-element grid [8]. This grid is constructed in such a way that the studied area consists only of finite elements - tetrahedron and meets the following requirements:

- The grid must be compatible, that is, in the three-dimensional case, the tetrahedron face can be divided only between two elements;
- The quality of the shape of the elements should be optimized to reduce the sampling error. In particular, degenerate elements are not allowed;
- The size and shape of the elements should correspond to the functions of the form of the corresponding nodes;
- The numbering of nodes and their orientation must be consistent;
- Some nodes, ribs, and faces can be created in the specified locations.

An algorithm based on the initial triangulation on a plane is implemented to sample a three-dimensional region. Based on the triangulation carried out on the plane, the three-dimensional grid is constructed by building up layers of similar triangulation, which was constructed on the initial layer. The formation of tetrahedra occurs by establishing links between triangular faces. To represent the discretization, the data structure “Tetrahedra-triangles-nodes” was selected.

For triangulation on the plane an iterative algorithm “remove and build” was selected on the basis of Delaunet's criterion [11]. Using the Delaunay triangulation algorithms in 3D it is not necessary to get a grid of tetrahedra, whose elements are correct for finite element analysis. To estimate the quality of the constructed tetrahedron it is necessary to get the value of the coefficient γ [13], which is determined by the relation volume of the tetrahedron to the sum of areas of its faces:

$$\gamma = \frac{12\sqrt{3}(AB \times AC) \cdot AD}{(AB^2 + BC^2 + CA^2 + AD^2 + BD^2 + CD^2)^{3/2}}$$

where *A,B,C,D* – peak of the tetrahedron.

The normalizing factor is the value of $12\sqrt{3}$, respectively, the maximum volume of the equilateral tetrahedron will be equal to 1. The tetrahedron, the coefficient γ for which will be equal to 0, are considered degenerate and subject to removal.

Software implementation

For designing the software, the main essence of the finite element method in the context of object-oriented programming is formulated. The software for finite element calculation is written in C # in the Microsoft Visual Studio environment using CUDA parallel computing technology and consists of three modules that are responsible for domain sampling, matrix generation, and equation solving. The features that are available to the user are shown in the usage diagram (Fig. 1).

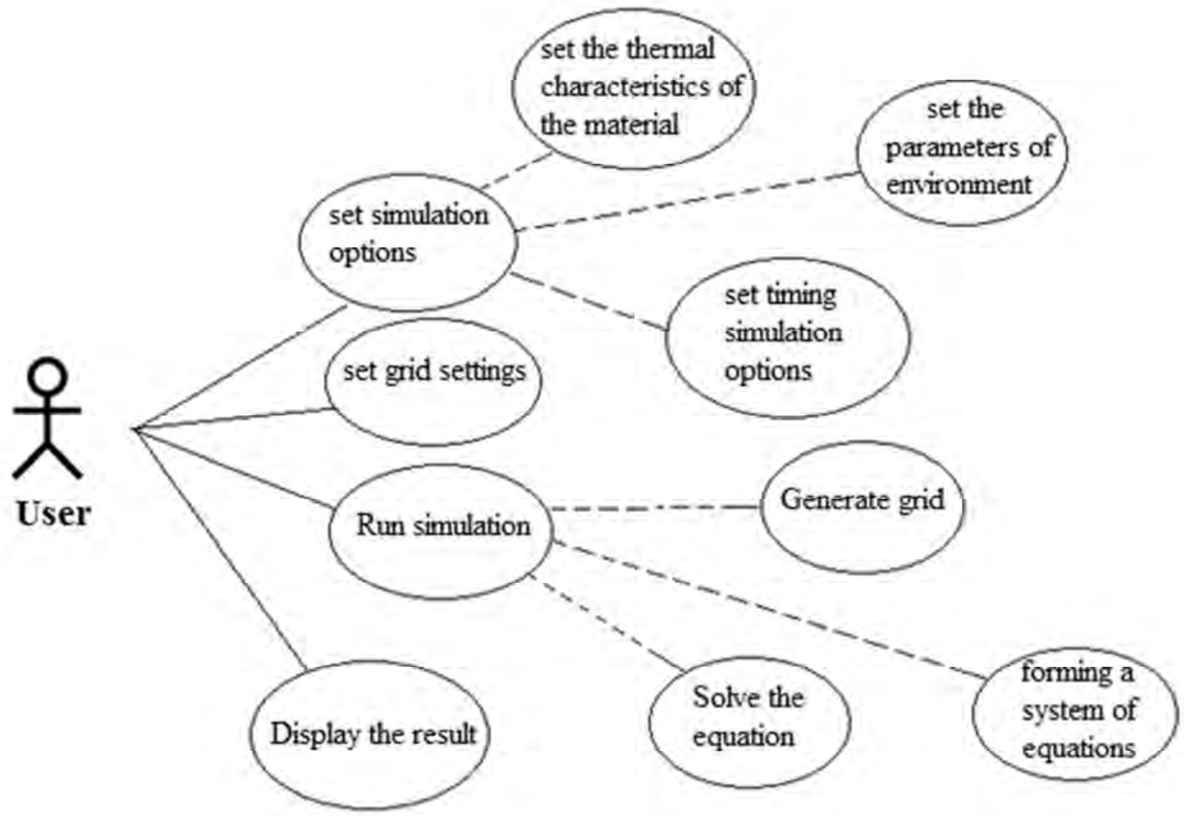

Fig. 1. Use case diagram

For the program realization of the partition of a predefined area for the problem of heat transfer and its reflection in a three-dimensional space, the design of the future structure of the system is presented, which is presented in the diagram of the classes (Fig. 2).

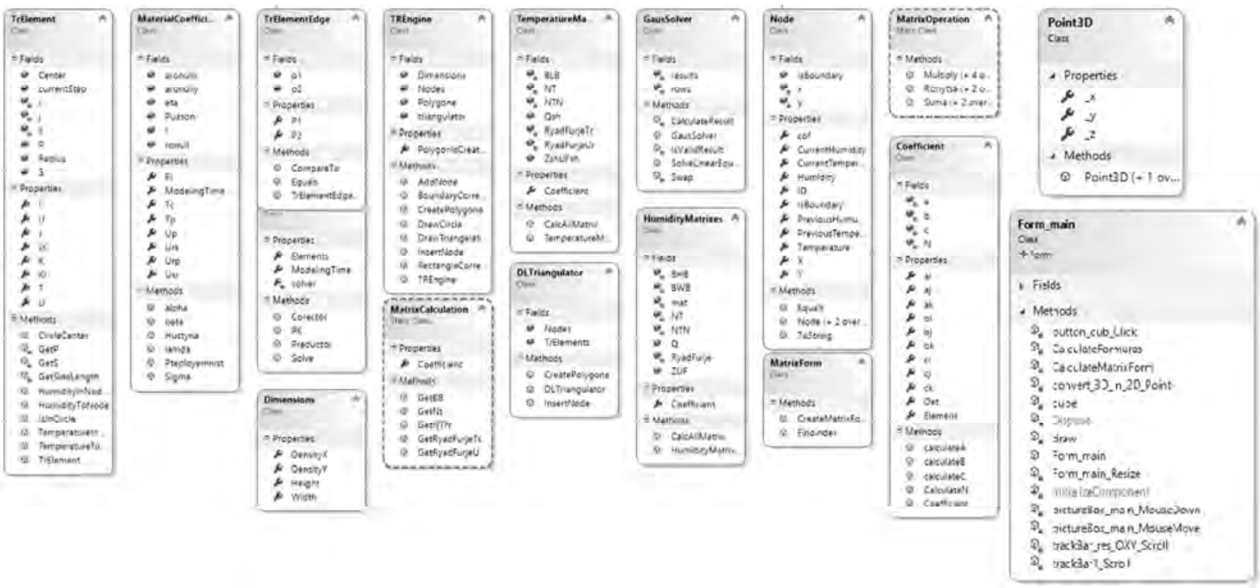

Fig. 2. Class Diagram

The classes responsible for the 3D partition of the area are displayed at the end of the diagram. The Point3D class contains constructors and methods that describe a point in 3D space. Formation of a point occurs in the class “Form_main”. To accumulate a pool of points, a collection of figure_3D has been created, which uses the draw method to form a k-d tree. The end result is a cube image implemented in “PictureBox”, which is divided into 6 tetrahedron.

Due to the fact that the realization of triangulation and its reflection is a rather laborious operation, the technology of parallel computing CUDA is used. The parallel computing platform CUDA provides a set of extensions for C # programming languages that allow you to parallelize tasks at the level of small and large structural units [17-18]. Therefore, CUDAfy [19–22] is used to program the C # language in the Visual Studio environment using GPGPU. A module that performs the conversion of input streams at the output is the kernel in the program is called “kernel”. Separate cores have ties with each other, which allow the formation of complex flow schemes for input data flows. The call to the module is from the body of the program, namely the method of processing the click of a button on the form “button_cub_Click”. The number of threads that are broken by the kernel method is determined by the draw method. In this case, the flows will be the same number as the lines used to create the image.

The fragmentation of the triangulation of the region using the parallel computing architecture is given in the following code:

```
      [Cudafy]
       public static void kernel(GThread thread)
          {   int tid = thread.blockIdx.x;
            for(int i = 0; i < val.Count - 1; i++)
         {//tetrahedral visualization
grf.DrawLine(pen_figure_3D,convert_3D_in_2D_Point(val[tid]),convert_3D_in_2D_Point(val[tid + 1]));
}}}}
    public static void draw()
      {//Connecting the Cudafy module
    CudafyModule km = CudafyTranslator.Cudafy();
    GPGPU gpu=
    CudafyHost.GetDevice(CudafyModes.Target,CudafyModes.DeviceId);
         gpu.LoadModule(km);
         if (figure_3D.Count <= 0)
```

```
            return;
        Bitmap bmp = new Bitmap(pictureBox_main.Width,pictureBox_main.Height);
        Graphics grf = Graphics.FromImage(bmp);
            Pen pen_figure_3D = new Pen(Color.Red);
            // launch add on val.Count threads
            gpu.Launch(val.Count, 1).kernel();}
            pictureBox_main.Image = bmp;
            pictureBox_main.Refresh();
            GC.Collect();}
```

The final image of the realized problem of partitioning the region on the tetrahedra is depicted in Fig. 3. Since the FEM is used for solving the heat transfer problem, for which a matrix of form is required, the program interface consists of two tabs, namely, "Sabstrate" and "Form matrix".

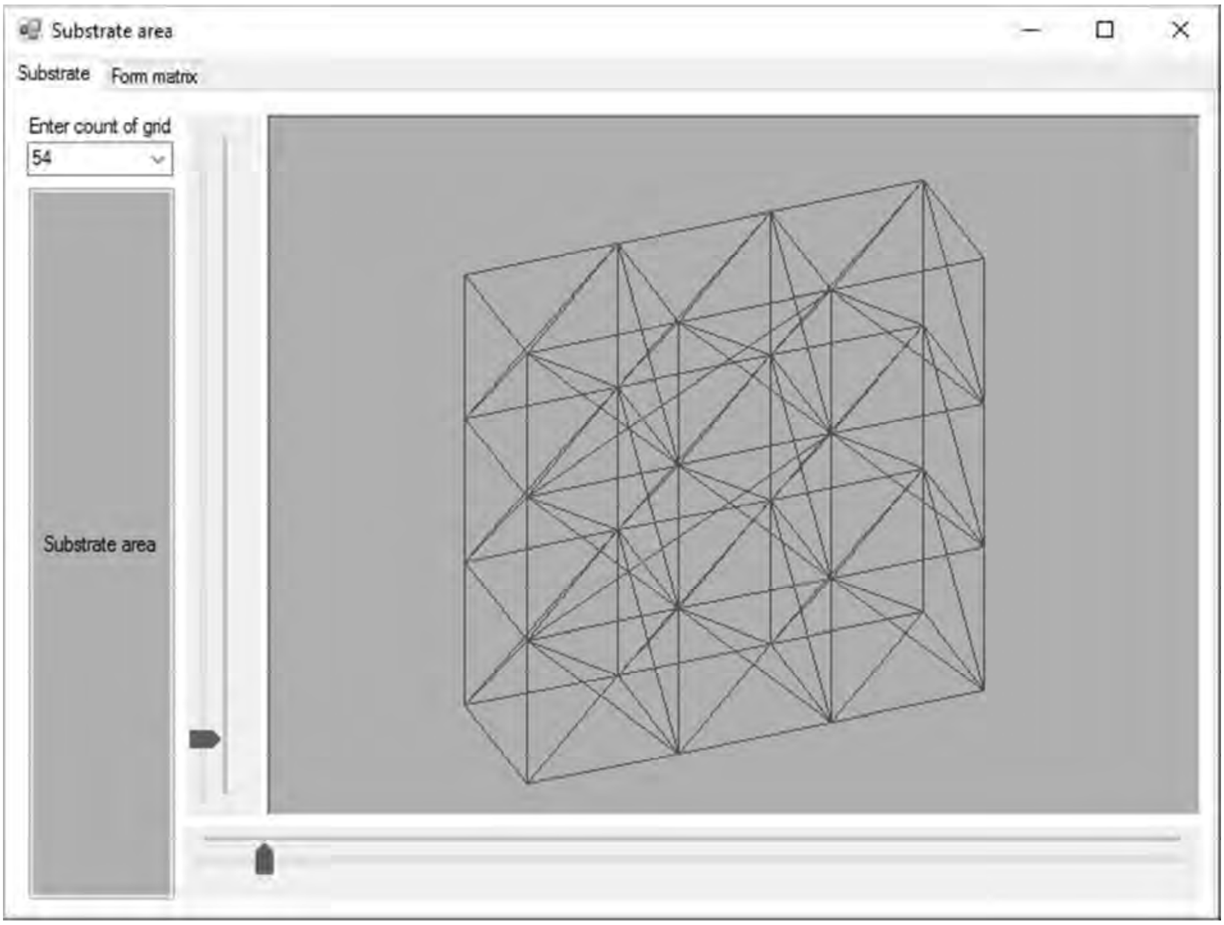

Fig. 3. Image sampling

Numerical experiment

Taking into account the mathematical model (1)–(4) and the software developed for it, numerical experiments were conducted to investigate the dynamics of spatial heat transfer in wood during drying. To do this, use the following environment settings: Tcep=80^0C , relative humidity φ=60 % Characteristics of pine wood: a_{m_x}= 0.97 cm^2/sec, a_{m_y}= 1.3 cm^2/sec, a_{m_z}= 1.8 cm^2/sec; coefficients of thermal conductivity and conductivity: α = 23Vt /$(m^2 \cdot K)$, β = $2 \cdot 10^{-6}$ м$/sec$. Other characteristics and necessary empirical dependencies are obtained on the basis of experimental data [15, 16]. In Fig.4, 5 shows the dynamics of the change in the moisture content $U^* = (U - U_0)/U_0$ for different values of drying time in wood ρ_0 = 450 kh/m^3.

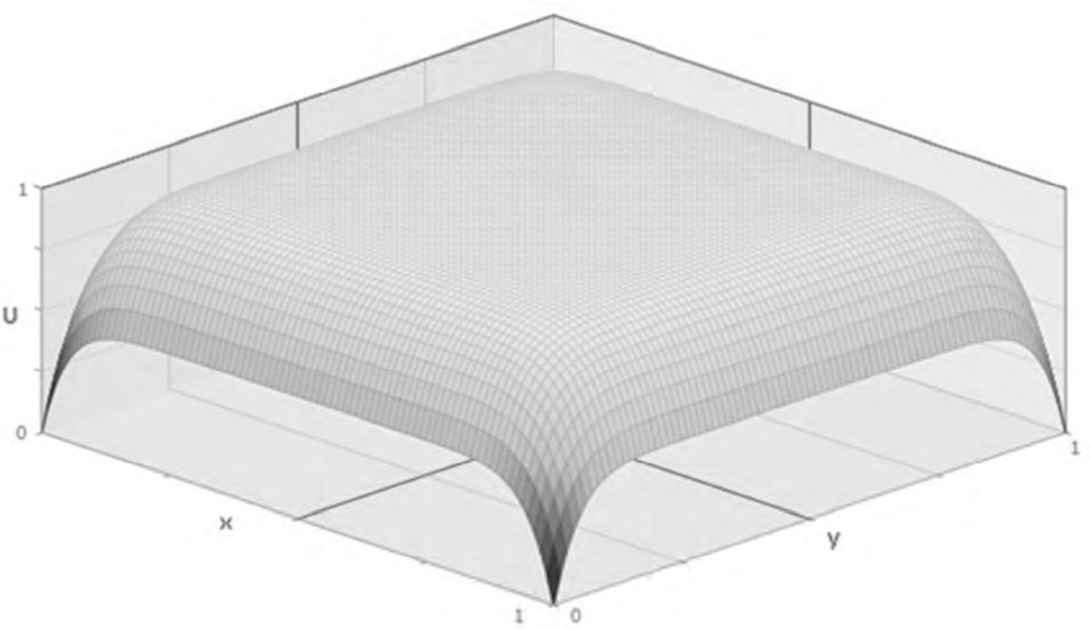

Fig. 4. Changes in moisture content for τ=2 hour ($R_1 = R_2; R_3 = 0$)

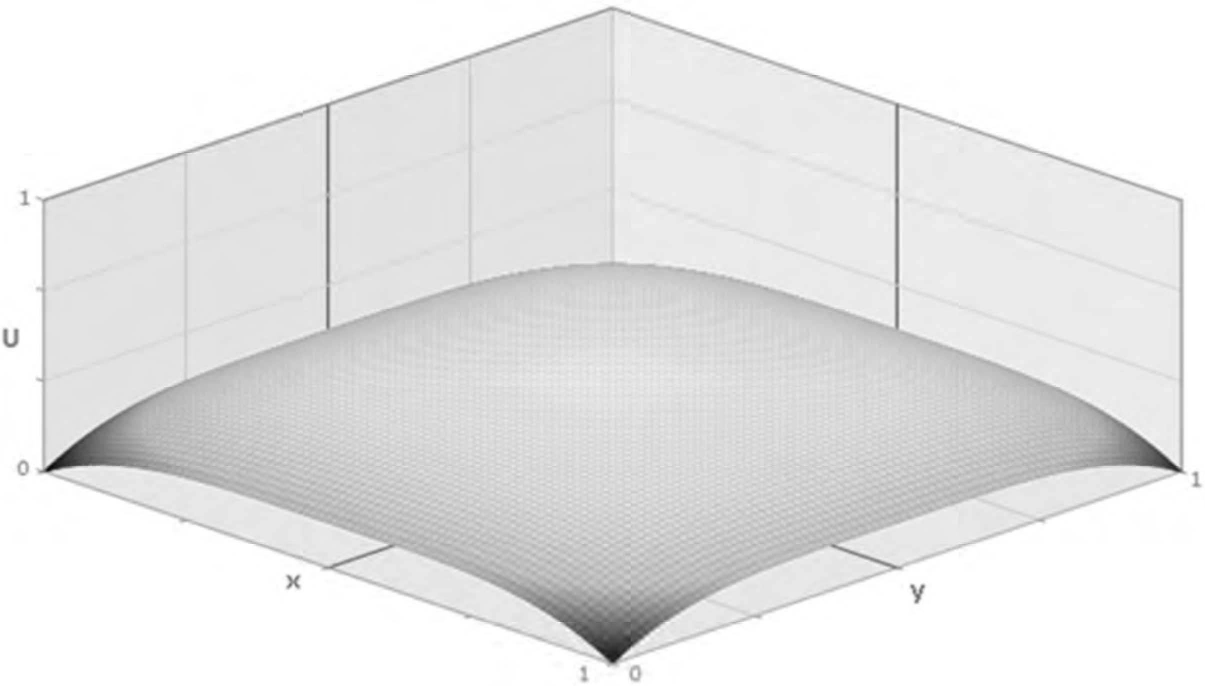

Fig. 5. Changes in moisture content for τ=40 hour ($R_1 = R_2; R_3 = 0$)

Conclusions

The mathematical model of heat-mass transfer in anisotropic capillary-porous materials in the form of a system of differential equations with initial and boundary conditions of the third kind is presented. To study the physical processes of connected heat transfer, software developed using CUDA parallel computing technologies has been developed.

A user friendly graphical interface was designed to approximate the finite-element splitting of the 3D domain. On the basis of numerical experiments, regularities of influence of geometric sizes, structural anisotropy, initial values of temperature, humidity and dynamics of spatial nonisothermal moisture transfer in wood were revealed.

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