The article deals with the proposed methods and models for planning of routes, based on in-depth study of passenger flows.

Keywords: passenger-transport system of the city, a study of passenger flows, routes planning.

Introduction
Development and improvement of public transport route network in urban areas remains urgent task. The dense buildings in downtown stipulates further development of peripheral neighborhoods on the outskirts. This exacerbates the problem of the transport connection in downtown and remote neighborhoods. Insufficient provision of neighborhoods by public transport means, leads to the transport disintegration of city and encourage the use of private cars. This, in turn, causes the obstruction of downtown and other active areas of the city by cars, that are parked for the day. A large number of routes, that connect the peripheral areas, normally pass through the downtown, causing it overloading by private cars and public transport. Steady tendency to downtowns glut by vehicles necessitates further research of passenger and traffic flows and improvement of public transport planning.

The aim is to develop methods and models for planning and improving public transport system, based on in-depth study of the city passenger flows.

The work is a further development of methods and approaches, proposed by author, and presented in [1–3].

Features of planning of city public transport
Public transport system, that ensuring the central part of the city and its peripheral areas, characterized by the following features:

– Remoteness of peripheral neighborhoods of the city from center increases the motion time and requires a larger number of vehicles on the route to provide the necessary interval of motion;
– The need of more routes for thro connection between neighborhoods of the city;
– Less, compared to the city center, the intensity of passenger flows for peripheral neighborhoods questioned the feasibility of using large buses and electric transport;
– The significant role of economic factors in the formation and operation of public transport routes;
– Citizens of low and middle income (which occupy the peripheral neighborhoods because of the lower cost of housing) are potential passengers of public transport;
– Large operating costs of private vehicle or its absence lead to the use of public transport for daily trips of many city inhabitants to places of work;
An obvious interest of passengers in ensuring of direct transport connections, to avoid double payment and reduce the time to changes of vehicle.

These features indicate the need for in-depth study of passenger flows and formalize the process of routes planning based on mathematical models.

**Models for the description of passenger flow and planning of city routes**

One of the main labor-intensive tasks, that need to be addressed when planning routes, is the collection and processing of reliable information about the actual passenger flows of the city.

There are the following three stages of the city route network formation:

1. Formation of routes to new areas, which is based on a sample survey and interrogation of potential passengers of public transport. Because of the relatively small sample size the data for forecasting the intensity of passenger flows are approximate in nature and suitable only for initial formation of routes.

2. Improving of routes, and partial optimization of the number of vehicles and traffic schedules, are based on a study of passenger flows in vehicles and characteristic section of the city [1]. Studies of real passenger flows, and their confirmed statistical stability, provides the formalization of routes improvement, based on mathematical models and computer simulation [2].

3. Planning and further improving existing city routes based on in-depth study of real passenger flows between all stops of the city. Gathering information about these passenger flows is also a time consuming task, but it provide the most detailed description for routes planning and optimization.

For inter-city and suburban routes specified information can be obtained from tickets to pay for travel. In cities, this problem some authors propose to solve with e-ticket, but it requires double validation of the ticket at the entrance and exit of passengers. Therefore, to collect the necessary information during inspections, in this paper proposed the using of coupons, that issued at the entrance to public transport and withdrawn at the exit. On the coupon indicates the name of the stop passenger boarding and at exit - coupons are collected in containers, with name of stop. The sample surveys are conducted in selected buses for all routes of the city to provide the necessary statistical data. This method ensures obtaining the necessary statistical information about passenger flows between stops of the city at acceptable labor costs. In detail, this method of collection and initial processing of statistical information is described in [3].

The approach to planning and improving of routes is considered for illustrative fragment of the route network, that presented in Fig. 1.

![Fig.1. Illustrative fragment of the city route network](image)

In this network there are such routes:
- m1: 1-3-5-6-7-10-7-6-5-3-1
- m2: 2-3-5-6-7-8-7-6-5-3-2
- m3: 4-5-6-7-9-7-6-5-4
- m4: 3-5-6-7-6-5-3
- m5: 4-5-6-7-8-7-6-5-4 (planned)

Unlike traditional approaches, based on an analysis of the number of passengers, as the main parameter, that describes passenger flows between pairs of stops \((i, j)\) for the bus \(n\) of route \(m\) during the time of the day \(t\), the speed of passengers influx \(V(m, n, i, j, t)\) is proposed

\[
V(m, n, i, j, t) = \frac{K(m, n, i, j, t)}{\Delta T(m, t)} \text{ (pass/min)}
\] (1)
Where: \( K(m, n, i, j, t) \) – the number of passengers, that traveling from stop \( i \) to stop \( j \) in bus \( n \) of route \( m \) at time \( t \); \( \Delta T(m, t) \) – the length of time (in minutes) between buses \( n-1 \) and \( n \) of route \( m \) at time \( t \) during the day.

The results of calculations \( V(m, n, i, j, t) \) for illustrative example are presented in Table 1.

**Table 1**

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This value \( V(m, n, i, j, t) \) does not depend on the change of passengers accumulation interval at the bus stop \( i \) of route \( m \). Accumulation interval determined as time interval between successive buses \( n-1 \) and \( n \) of route \( m \) at time \( t \) during the day. The value of \( V(m, i, j, t) \) for arbitrary points between the times of sample is determined by interpolation.

Graph, that describes the change of passengers influx speed between stops \( i \) and \( j \) for \( m1 \) and \( m2 \) routes during the day, is presented in Fig. 2.

By summation of value \( V(m, i, j, t) \) for all city routes \( M \) can get the statistical information that describes the speed of passenger inflow between all city stops for sample values \( t \) during the day.

\[
V(M, i, j, t) = \sum_{m} V(m, i, j, t)
\]

Where: \( m = 1, M; M \) – total number of routes.

The change of the total passengers inflow speed \( V(M, i, j, t) \) between a pair of stops \((i, j)\) over time during the day, is defined on the basis of examination of all routes \( M \) and presented in Fig. 3.
The dependence of \( V(M, i, j, t) \) is defined for all pairs of city stops \((i, j), i = 1, Z; j = 1, Z\), where: \( Z \) – total number of stops. Based on this dependence, for a given time \( t \) determined the all values of \( V(M, i, j, t) \), that can be represented as a matrix.

![Fig. 2. The change of passengers inflow speed between stops \( i \) and \( j \) for routes \( m_1 \) and \( m_2 \) during the day](image)

![Fig. 3. The change of the total passengers inflow speed between stops \( i \) and \( j \) for all routes during the day](image)

**The third stage of routes planning**

The purpose of the third stage of routes planning is the redistribution of passenger flows between existing and planned routes for further optimization of the number of vehicles and charting their movements.

The proposed planning method comprises the following steps:

1. To calculate the required number of buses on routes, the total speed of passengers inflow \( V(M, i, j, t) \) between all pairs of city stops for a morning peak time \( t = 8.00 \) is determined.

2. The speed of passengers inflow for pairs of stops on monopoly areas of the route is determined. Monopoly called the areas, for which thro connection between a pair of stops \((i, j)\) may be carried out only one route:

\[
V(m_1,1,3,t), \ V(m_1,1,5,t), \ V(m_1,1,6,t), \ V(m_1,1,7,t), \ V(m_1,1,10,t), \ V(m_1,10,7,t), \ V(m_1,10,6,t), \ V(m_1,10,5,t), \ V(m_1,10,3,t), \ V(m_1,10,1,t);
\]

\[
V(m_2,n,i,j,t)
\]
For each route \( m \) the maximum total speed of passengers inflow \( V_{\text{max}}(m, t) \), for pairs of stops on monopoly areas, is determined:

\[
V_{(m1,1,3,t)} = V(m1,1,3,t) + V(m1,1,5,t) + V(m1,1,6,t) + V(m1,1,7,t) + V(m1,1,10,t);
V_{(m1,1,7,10,t)} = V(m1,1,10,t) + V(m1,3,10,t) + V(m1,5,10,t) + V(m1,6,10,t) + V(m1,7,10,t);
V_{(m1,3,1,t)} = V(m1,10,1,t) + V(m1,7,1,t) + V(m1,6,1,t) + V(m1,5,1,t) + V(m1,3,1,t);
V_{\text{max}(m1,t)} = \max(V_{(m1,1,3,t)}, V_{(m1,7,10,t)}, V_{(m1,3,1,t)});
\]

\[
V_{(m2,2,3,t)} = V(m2,2,3,t) + V(m2,2,5,t) + V(m2,2,6,t) + V(m2,2,7,t) + V(m2,2,8,t);
V_{(m2,3,2,t)} = V(m2,8,2,t) + V(m2,7,2,t) + V(m2,6,2,t) + V(m2,5,2,t) + V(m2,3,2,t);
V_{\text{max}(m2,t)} = \max(V_{(m2,2,3,t)}, V_{(m2,3,2,t)});
\]

\[
V_{(m3,3,9,t)} = V(m3,4,9,t) + V(m3,5,9,t) + V(m3,6,9,t) + V(m3,7,9,t);
V_{(m3,9,7,t)} = V(m3,9,7,t) + V(m3,9,6,t) + V(m3,9,5,t) + V(m3,9,4,t);
V_{\text{max}(m3,t)} = \max(V_{(m3,3,9,t)}, V_{(m3,9,7,t)});
V_{(m5,4,8,t)} = V(m5,4,8,t);
V_{(m5,8,4,t)} = V(m5,8,4,t);
V_{\text{max}(m5,t)} = \max(V_{(m5,4,8,t)}, V_{(m5,8,4,t)});
\]

4. This maximum speed of passengers inflow determines the required transportation capacity
\( W(m, t) \), which should provide a buses of route \( V_{\text{max}}(m, t) \leq W(m, t) \). The maximum transportation capacity \( W(m, t) \) of buses on route \( m \) with interval \( \Delta T(m, t) \) in peak traffic period is defined as follows:

\[
W(m, t) = P(m, t) / \Delta T(m, t) \text{ (pass/min)}
\]

Where: \( P(m, t) \) - capacity of the bus \( (P(m, t) = 20, 40, 80, 100, 120, 150) \).

5. For a given transportation capacity \( W(m, t) = V_{\text{max}}(m, t) \), the bus capacity \( P(m, t) \) is selected and the necessary interval of motion \( \Delta T(m, t) \) is determined

\[
\Delta T(m, t) = W(m, t) / P(m, t)
\]

6. For all other (non-monopoly) pairs of stops \( (i, j) \) of route \( m \) residual total speed of passengers inflow \( V_z(M, i, j, t) \) is defined as

\[
V_z(M, i, j, t) = V(M, i, j, t) - W(m, t)
\]

7. Redistribution of residual total speed of passenger inflow for non-monopoly pairs of stops is based on the analysis and correction of buses transport capacity for overlapping areas of the routes, or the creation of additional routes.

8. Based on the required interval of motion \( \Delta T(m, t) \) and the cycle time \( T(m, t) \), the required number of buses \( N(m) \) for each route \( m \) is determined

\[
N(m) = T(m, t) / \Delta T(m, t)
\]

Conclusions

The presented methods and models, based on in-depth study of passenger flows, provide formalization and improving of the routes planning.