Determination of Stokes coefficients according to the GOCE satellite
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The last achievements of the satellite geodesy is the project of European Space Agency – the GOCE satellite mission (Gravity of field and steady – state of Ocean Circulation Explorer) which is using the satellite gradiometry method. The gravitational field of the Earth is represented traditionally as a finite series of spherical harmonic functions of the gravitational field model of the finite number of parameters Cnm, Snm. The corresponding coefficients Cnm, Snm were derived based on the second Neumann method including the construction of Gauss-Legendre grid.

Key words: satellite gradiometry, quadrature formulas, GOCE, harmonic coefficients Cnm, Snm

I. Introduction

The Earth’s gravity field reflects the mass distribution and its transport in the Earth’s interior and on its surface. In 2000, the era of dedicated satellite gravity missions began with the launch of CHAMP (CHAllenging Minisatellite Payload), followed by the launches of GRACE (Gravity Recovery And Climate Experiment) in 2002, and GOCE (Gravity field and steady-state Ocean Circulation Explorer) in 2009. Based on data of these missions, global Earth’s gravity field models with homogeneous accuracy and increasingly high spatial resolution could be derived.

It is convenient to present Earth's gravity field by the row of the selected base functions, i.e. for modelling of all violations of the gravity field, an endless number of parameters, so-called base coefficients, is needed. At present these unknown can be determined from the observation of the gravity field, namely by the methods of satellite gradiometry.

The Earth’s gravitational potential \( V \) is usually parameterized in terms of coefficients of a spherical harmonic series expansion in spherical coordinates

\[
V(r, \theta, \lambda) = \frac{GM}{r} \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} \left( \frac{a}{r} \right)^{l+1} \left[ C_{lm} \cos m\lambda + S_{lm} \sin m\lambda \right] P_{lm}(\cos \theta)
\]

with \( G \) the gravitational constant, \( M \) mass of the Earth, \( a \) mean Earth radius, \( P_{lm} \) the fully normalized Legendre polynomials of degree \( n \) and order \( m \), and \( C_{lm}, S_{lm} \) the corresponding coefficients to be estimated. All observation types are functionals of the gravitational potential \( V \).

Given the large number (about 81 million) output as opposed to the traditional methods of constructing models of the gravitational field of the Earth concentrate their attention on the so-called second- Neumann method, which applies certain weights that preserves orthogonality of Legendre functions. This leads to the use of the most accurate quadrature formulas of Gauss-Legendre at the appropriate grid.

Initial data taken data gradiometers satellite GOCE EGG TRF_2 (gravity gradients in the system LNOF, their accuracy and geographic coordinates \( \varphi, \lambda, r \) for a period of about 3 years. Using radial gradients \( V_{zz} \), we can calculate the harmonic coefficients of the gravitational field of the Earth.

After the rejection process data of data received is as follows in Table 1

<table>
<thead>
<tr>
<th>DATA SET</th>
<th>to rejection</th>
<th>after rejection</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>80 860 570</td>
<td>80 849 326</td>
</tr>
</tbody>
</table>

In constructing models for large amounts of data in all cases, the main drawback is the fact that the calculation takes time. Therefore, the work of filtering applied by the fast Fourier transform (FFT)

II. Second Neumann method

Neumann showed that if one chooses the latitude circles to coincide with the zeros of the Legendre polynomial of degree \( L+1 \), i.e.

\[
P_{L+1}(x_i) = 0, \quad \forall i = 1, 2, ..., L+1
\]

the number of parallels can be reduced to \( N = L+1 \). The order of precision of the numerical quadrature is doubled, and eq. remains true with half the number of parallels as compared with the first method. The ratio of function values versus harmonic coefficients will roughly become two. The arrangement of points on the sphere at the nodes of a Gaussian grid is strongly connected to the Gauss-Legendre quadrature. The grid features equiangular spacing along \( L \) circles of latitude with

\[
\Delta \lambda = \frac{\pi}{L} \Rightarrow \lambda_i = \frac{\Delta \lambda}{2} + i \cdot \Delta \lambda
\]

\[0 \leq i \leq 2L\]

Along the meridians the points are located at \( L \) parallels at the L zeros of the Legendre polynomial of degree L, \( P_l(\cos \theta) = 0 \)

Consequently, the number of grid points sums up to

\[I = 2 \cdot L^2\]

The Gauss grid looks quite similar to the corresponding geographical grid with the same number of parallels. Its distinctive feature is the unique choice of the location of the circles of latitude.

The Gauss-Legendre quadrature allows the recovery of a spherical harmonic expansion of degree \( N = L-1 \) from only \( L \) circles of latitude. On the other hand, the parallels cannot be chosen arbitrarily, but have to be located along the zeros of the Legendre polynomial of degree L. Therefore, the quadrature nodes of the Gauss-Legendre quadrature method coincide with the grid points of the Gaussian grid. The determination of the spherical harmonic coefficients can be split into a two-step procedure:

\[
A_n(\theta) = \frac{1}{2L(l+1)} \sum_{l=0}^{2L-1} V_{zz}(\theta, \lambda_i) \left( \cos m\lambda_i \right) \sin m\lambda_i
\]

\[
B_n(\theta) = \frac{1}{2L(l+1)} \sum_{l=0}^{2L-1} \sum_{m=0}^{l} w_{lm} P_{lm}(\cos \theta) \left( \cos m\lambda_i \right) \sin m\lambda_i
\]
So based approach, a model of harmonic coefficients according to the order of 250 - GOCE - LP_GOCE-01s. Compared with the model EGM2008 a model converges to 220 order (Figure 1).

Using the inverse Stokes formulas, we can construct a map of the region of the Black Sea using our model LP_GOCE-01s (Figure 2).

\[ \Delta g(\theta, \lambda, r) = \frac{1}{4\pi R} \int \int T(\theta', \lambda', R) Z(\Psi, r) d\sigma \]  

**Conclusion**

For the first time instead of the classical methods for constructing gravitational field of the Earth the method of direct calculation of harmonic coefficients by satellite gradiometry.

In models EGM2008 LP_GOCE-01s and under (Figure 1) of the order of 200 to 250 is a little difference in power dispersion may be due to the fact that the model LP.GOCE-01s determined only satellite data, and also EGM2008 determined and surface data.

References


