

## MATHEMATICAL SIMULATION OF HIGH-TEMPERATURE DRYING OF WOOD

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Здійснено математичне моделювання процесів тепломасоперенесення і в'язкопружного деформування у гігроскопічних капілярно-пористих матеріалах зі змінними анізотропними тепломеханічними характеристиками, що має важливе значення для раціонального вибору та обґрунтування енергозберіжних технологій високотемпературного сушіння деревини за умови забезпечення необхідної якості продукції. Зреалізовано сформульовану математичну модель деформування деревини під час сушіння, яка дає змогу визначити двовимірний напружено-деформівний стан в умовах дифузійно-фільтраційного тепловологоволовоперенесення, методом скінченних елементів. Побудовано алгоритм методу скінченних елементів для дослідження двовимірного анізотропного напружено-деформівного стану під час сушіння капілярно-пористих матеріалів у в'язкопружній області деформування. У результаті обчислювальних експериментів встановлено закономірності впливу анізотропії теплофізичних та механічних характеристик деревини, її початкової вологості, геометричних параметрів і характеристик агента сушіння на зміну двовимірного температурно-вологісного та в'язкопружного стану деревини під час конвективного сушіння.

**Ключові слова:** математична модель, метод скінченних елементів, тепломасоперенесення, в'язкопружний стан, конвективне сушіння, деревина.

The article implemented mathematical modelling of processes of heat-mass-transfer and viscous-elastic deformation in in hygroscopic capillary-porous materials with variable anisotropic heat and mechanics characteristics what is of importance for the rational choice and substantiation of energy conservative technologies of timber drying under the conditions of necessary qualitative production providing. There has been implemented the formulated mathematical model of timber deformation during the drying process which enables to identify two-dimensional intense-deforming state under the conditions of diffusion-filtration heat and moisture transfer of the finite elements method. This method has been developed for the research of the two-dimensional anisotropic intense-deforming state during the capillary-porous materials drying process in an viscous-elastic area of deformation. As a result of calculable experiments regularity of anisotropy influence of thermo-physical and mechanical descriptions of wood were set it initial humidity on changing of two-dimensional temperature-humidity and viscous-elastic state of wood during the convective drying.

**Key words:** mathematical model, the method of finite elements, heat and mass transfer, viscous-elastic state, convective drying process, timber.

### Actuality of research

Drying capillary-porous materials are widely used in various industries and require significant energy, time and material costs. Therefore, increasing the energo-resources under the condition of maintaining the required quality of products is an important task. One of the approaches to intensify the process of drying is to use high-temperature regimes. For them, one of the defining factors of influence on the process of removal of moisture is the gradient of the internal residual pressure of vapor-air mixture. In

this respect of solving this problem is to develop mathematical models and methods in the study of diffusion-filtration processes of heat-mass-transfer and deformation of capillary-porous materials in drying process, which give the opportunity to detail the technology of drying, to predict the quality of the finished products and increase the efficiency of the development of equipment to provide the desired performance. This problem is compounded by the fact that wood belongs to the class of the hydroficated physically nonlinear polymers, which are characterized by considerable variability of structural and mechanical properties.

### Analysis of existing results

Today is constructed two-dimensional mathematical model of deformation of the wood of rheological behavior of [1, 2, 3]. The application of numerical methods allowed to explore two-dimensional mathematical model is not isothermal moisture-transfer and stress-deforming condition of wood during drying of viscous-elastic behavior of wood and anisotropy variables of thermo-mechanical characteristics [4, 5]. The study of the processes of regeneration of moisture transfer for high-intensive process of drying of capillary-porous materials devoted to [6, 7]. However, the practical use of these solutions of differential equations, molar-molecular transfer is difficult in the absence of reliable data coefficients of molar transfer of moisture. Features of mathematical modeling of influence of pressure on the processes of heat and mass transfer dedicated to work [8–10].

The analysis in the field of mathematical modeling of high-temperature process of drying of capillary-porous materials indicate different approaches regarding the origin of the main potential regeneration of heat and mass transfer and its dependence on temperature and moisture. Existing mathematical models do not take into account the entire complex of processes of regeneration of heat and mass transfer and deformation during the drying of capillary-porous materials. Because the construction and analysis of mathematical models of diffusion-filtration heat and moisture transfer in capillary-porous materials in drying process taking into account the peculiarities of technological factors is relevant task. Development program-algorithmic means implementation of these models allows you to predict the dynamics of transfer potentials at different stages of heat and mass exchange processes, as well as the development of effective technological modes of drying process.

### Mathematical model of diffusion-filtration heat and moist transfer

For the modeling of non-stationary fields of diffusion-filtration moisture transfer using a system of differential equations [6]:

$$C_q \frac{\partial T}{\partial t} = K_{11} \nabla^2 T + K_{12} \nabla^2 U + K_{13} \nabla^2 P; \quad (1)$$

$$C_m \frac{\partial U}{\partial t} = K_{21} \nabla^2 T + K_{22} \nabla^2 U + K_{23} \nabla^2 P; \quad (2)$$

$$C_p \frac{\partial P}{\partial t} = K_{31} \nabla^2 T + K_{32} \nabla^2 U + K_{33} \nabla^2 P. \quad (3)$$

Where the working variables are temperature  $T(x, y, t)$ , moisture potential  $U(x, y, t)$  and pressure  $P(x, y, t)$ . The coefficients  $C_q$ ,  $C_m$ ,  $C_p$ ,  $K_{11}$ ,  $K_{12}$ ,  $K_{13}$ ,  $K_{21}$ ,  $K_{22}$ ,  $K_{23}$ ,  $K_{31}$ ,  $K_{32}$ ,  $K_{33}$  are defined as

$$\begin{aligned} C_q &= \rho_0 c_q \delta / c_m; & C_m &= \varepsilon \lambda \rho_0 c_m; & C_p &= -\lambda \rho_0 c_p k_p / k_m; \\ K_{11} &= (k_q + \varepsilon \lambda k_m) \delta / c_m; & K_{12} &= \varepsilon \lambda k_m \delta / c_m; & K_{13} &= \varepsilon \lambda k_p \delta / c_m; \\ K_{21} &= \varepsilon \lambda k_m \delta / c_m; & K_{22} &= \varepsilon \lambda k_m; & K_{23} &= \varepsilon \lambda k_p; \\ K_{31} &= \varepsilon \lambda k_p \delta / c_m; & K_{32} &= \varepsilon \lambda k_p; & K_{33} &= -\lambda (1 - \varepsilon) k_p^2 / k_m. \end{aligned}$$

Where  $C_m$  – specific moisture capacity,  $C_p$  – air capacity,  $C_q$  – heat capacity,  $k_m$  – coefficient of moisture conductivity,  $k_p$  – moisture filtration coefficient,  $k_q$  – thermal conductivity,  $t$  – time,  $\rho_0$  – dry body density,  $\delta$  – thermo-gradient coefficient,  $\lambda$  – latent heat,  $\varepsilon$  – ratio of vapor diffusion coefficient to the coefficient of total moisture diffusion.

The boundary conditions associated with the system of equations (1) – (3) are given by

$$(k_q + \varepsilon \lambda \rho_0 \delta a_m) \frac{\partial T}{\partial n} + A_q(T - T_0) + A_\varepsilon(U - U_0) + (k_q + \varepsilon \lambda \rho_0 \delta a_m) / k_q = 0; \quad (4)$$

$$a_m \frac{\partial U}{\partial n} + A_\delta(T - T_0) + A_m(U - U_0) + (j_m / \rho_0 - a_m \delta j_a) / k_q = 0. \quad (5)$$

Where coefficients  $A_q$ ,  $A_\varepsilon$ ,  $A_\delta$ ,  $A_m$  are

$$A_q = \frac{(k_q + \varepsilon \lambda \rho_0 \delta a_m) \alpha_q}{k_q}; \quad A_\delta = -\frac{a_m \delta \alpha_q}{k_q};$$

$$A_\varepsilon = \frac{\lambda \rho_0 \alpha_m}{k_q} (1 - \varepsilon) (k_q + \varepsilon \lambda \rho_0 \delta a_m); \quad A_m = \alpha_m - \frac{a_m \alpha_m \rho_0 \lambda \delta}{k_q} (1 - \varepsilon).$$

Where  $\alpha_m$  is convective mass transfer coefficient,  $\alpha_q$  is convective heat transfer coefficient.

The initial conditions have the form

$$T(x, y, t) = T_0(x, y); \quad U(x, y, t) = U_0(x, y); \quad P(x, y, t) = P_0(x, y); \quad (6)$$

where  $T_0$ ,  $U_0$ ,  $P_0$  – values of temperature, moisture, and pressure at the beginning of the drying process.

### Numerical realization of mathematical model

The governing differential equations (1)–(3) were transformed into element equations using the Galerkin's weighted residual method [11]. The working variables T, U and P were approximated in terms of the respective nodal values  $T_j$ ,  $U_j$  and  $P_j$  by interpolating functions:

$$\bar{T} = \sum_{j=1}^n N_j(x, y) T_j(t); \quad \bar{U} = \sum_{j=1}^n N_j(x, y) U_j(t); \quad \bar{P} = \sum_{j=1}^n N_j(x, y) P_j(t); \quad (7)$$

Where  $N_j$  is the weighting function, and  $\bar{T}$ ,  $\bar{U}$ ,  $\bar{P}$  are the average element values of temperature, moisture potential and pressure respectively.

Using the Galerkin weighted residual method and setting the residual of the weighted errors to zero, Eqs. (1) – (3) become

$$\int_{\Omega} N_i [\nabla \cdot (K_{11} \nabla \bar{T}) + \nabla \cdot (K_{12} \nabla \bar{U}) + \nabla \cdot (K_{13} \nabla \bar{P}) - C_q \frac{\partial \bar{T}}{\partial t}] d\Omega = 0 \quad (8)$$

$$\int_{\Omega} N_i [\nabla \cdot (K_{21} \nabla \bar{T}) + \nabla \cdot (K_{22} \nabla \bar{U}) + \nabla \cdot (K_{23} \nabla \bar{P}) - C_m \frac{\partial \bar{U}}{\partial t}] d\Omega = 0 \quad (9)$$

$$\int_{\Omega} N_i [\nabla \cdot (K_{31} \nabla \bar{T}) + \nabla \cdot (K_{32} \nabla \bar{U}) + \nabla \cdot (K_{33} \nabla \bar{P}) - C_p \frac{\partial \bar{P}}{\partial t}] d\Omega = 0 \quad (10)$$

For numeral realization of mathematical models of connected with each other processes of heat-mass transfer during wood drying (1)–(6) the Finite element method (FEM) is used [11]. Equivalent variation formulation of model is for this purpose got with assumption, that the change of moisture content in time is possible to show as a sum of constituents, related to the stream of mass transfer by the gradient of moisture content and temperature. Eventual system of equalizations for realization of mathematical model (7)–(8) after FEM looks like:

$$[C] \frac{\partial \{U\}}{\partial \tau} + [K] \{U\} + \{F\} = 0; \quad [\bar{C}] \frac{\partial \{T\}}{\partial \tau} + [\bar{K}] \{T\} + \{\bar{F}\} = 0; \quad [\bar{C}] \frac{\partial \{P\}}{\partial \tau} + [\bar{K}] \{P\} + \{\bar{F}\} = 0,$$

where:  $[C] = \int_V \rho_0 [N]^T [N] dV$ ;  $[K] = \int_V [B]^T [D^*] [B] dV + \int_S \rho_0 \beta_w [N]^T [N] dS$ ;  $\{F\} = \int_V [B]^T [H] [B] [T] dV - \int_S \rho_0 \beta U_p [N]^T dS$  – according to the matrix of thermo-physical properties of material, damping and

loading,  $\{N\}$  – matrix of form functions. Analogical matrixes are those  $[\overline{C}], [\overline{K}], \{\overline{F}\}, [\overline{C}], [\overline{K}], \{\overline{F}\}$ , related with the coefficient of heat conductivity and heat exchange.

For the values of change of temperature  $\{T\}$  and moisture  $\{U\}$  in time the Finite difference method is used. Then numeral realization of mathematical model (7)–(8) is taken to the decision of the kind of system equalizations:

$$[A]\{U\}_{next} = \{R\}; \quad [A_T]\{T\}_{next} = \{R_T\}; \quad [A_P]\{P\}_{next} = \{R_P\}; \quad (11)$$

As thermo-physical descriptions of wood depend on a temperature and moisture, and equalization of model (7)–(8) are related with each other, the iteration process of equalizations realization (10) is carried out on every sentinel step taking into account additional iteration procedure, which specifies influence of moisture on apportionment of temperature in material and vice versa. Completing of iterations for equalizations (10) foresees implementation of conditions:

$$\{U_n\} - \{U_{n-1}\} \leq 10^{-4}; \quad \{T_n\} - \{T_{n-1}\} \leq 10^{-4}. \quad \{P_n\} - \{P_{n-1}\} \leq 10^{-4}.$$

Mathematical modeling of deformation and stress in wood during drying. System of model equalizations for determination component of deformation vector  $\varepsilon = (\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12})^T$ , strains  $\sigma = (\sigma_{11}, \sigma_{22}, \sigma_{12})^T$ , temperature  $T(X, \tau)$ , and moisture content  $U(X, \tau)$  during drying wooden bar during time  $\tau \in [0, \tau_{dry}]$  in the area of cross-cut section  $\Omega = \{X = (x_1, x_2); x_1 \in [0, l_1], x_2 \in [0, l_2]\}$  the center of which is combined with beginning of co-ordinates, and the axes of anisotropy coincide with co-ordinate axes, it is built so. The components of tensions vector are satisfied by equalizations of equilibrium with boundary conditions which take into account absence of external efforts:

$$\mathbf{B}^T \sigma = 0. \quad (12)$$

Boundary conditions (that take into account the symmetry of the task region  $\Omega$ ) are:

$$u_i|_{x_i=0} = 0; \quad (13)$$

$$\sigma_{ii}|_{x_i=l_i} = 0, \quad (14)$$

where  $l_1, l_2$  are geometric dimensions.

Here are set notations:  $\sigma = (\sigma_{11}, \sigma_{22}, \sigma_{12})^T$  – strains component vector,  $\mathbf{B}$  – matrix of differential operators

$$\mathbf{B}^T = \begin{bmatrix} \frac{\partial}{\partial x_1} & 0 & \frac{\partial}{\partial x_2} \\ 0 & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_1} \end{bmatrix}.$$

Correlation between movements and vector of deformation  $\varepsilon = (\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12})^T$  is written this way:

$$\varepsilon = \mathbf{B}u, \quad (15)$$

where  $u$  is displacement vector components  $u = (u_1, u_2)^T$

For describing deformation processes in viscous elastic bodies, to which concerns wood, was used hereditary theory of elasticity [9]. It's correlation describes connection between components of strains and deformations in wood drying process, that is in tensorial form with the help of formula

$$\sigma(t) = C(\varepsilon - \varepsilon_T) - C \int_0^t R(t, \tau) \varepsilon(\tau) d\tau, \quad (16)$$

or in scalar form with the help of formulas:

$$\begin{aligned} \sigma_{11}(t) &= C_{11}(\varepsilon_{11} - \varepsilon_{T1}) - C_{11} \int_0^t R_{11}(t, \tau) \varepsilon_{11}(\tau) d\tau + C_{12}(\varepsilon_{22} - \varepsilon_{T2}) - C_{12} \int_0^t R_{12}(t, \tau) \varepsilon_{22}(\tau) d\tau; \\ \sigma_{22}(t) &= C_{21}(\varepsilon_{11} - \varepsilon_{T1}) - C_{21} \int_0^t R_{21}(t, \tau) \varepsilon_{11}(\tau) d\tau + C_{22}(\varepsilon_{22} - \varepsilon_{T2}) - C_{22} \int_0^t R_{22}(t, \tau) \varepsilon_{22}(\tau) d\tau; \\ \sigma_{12}(t) &= 2C_{33}(\varepsilon_{12} - \varepsilon_{T3}) - 2C_{33} \int_0^t R_{33}(t, \tau) \varepsilon_{12}(\tau) d\tau, \end{aligned} \quad (17)$$

where  $\varepsilon_T = \begin{bmatrix} \varepsilon_{T1} \\ \varepsilon_{T2} \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha_1 \Delta T + \beta_1 \Delta U \\ \alpha_2 \Delta T + \beta_2 \Delta U \\ 0 \end{bmatrix}$  – deformations vector, that caused by changeable gradients of

temperature  $\Delta T$  and moisture content  $\Delta U$  accordingly. Exactly these deformations are the main source of strains formation in wood during drying process. For anisotropic body in case of plain stressed-strained state, elasticity matrix is given by

$$\mathbf{C} = \begin{bmatrix} \frac{E_{11}}{1-\nu_1\nu_2} & \frac{\nu_1 E_{22}}{1-\nu_1\nu_2} & 0 \\ \frac{\nu_1 E_{22}}{1-\nu_1\nu_2} & \frac{E_{22}}{1-\nu_1\nu_2} & 0 \\ 0 & 0 & \mu \end{bmatrix}. \quad (18)$$

Here  $E_{11}$ ,  $E_{22}$  – modulus of elasticity,  $\nu_1$ ,  $\nu_2$  – Poisson coefficients,  $\mu$  – modulus of rigidity.

In this task was taken into account that elasticity matrix coefficients depend on temperature's value and on moisture content of material. Functions rheological behavior of wood during drying with regard to the mechanism of accumulation of irreversible strains are selected as

$$\varepsilon^*(\tau) = \left[ a_0 - \sum_{i=1}^M a_i \exp(-b_i \tau) \right] h(\tau) h(\tau_0 - \tau) - \left[ a_0 - \sum_{i=1}^M \alpha_i \exp(-\beta_i (\tau - \tau_0)) \right] h(\tau - \tau_0), \quad (19)$$

where  $h(\tau)$  – Heaviside function, and the unknown coefficients  $a_i$ ,  $b_i$ ,  $\alpha_i$ ,  $\beta_i$  determined by least squares approximation based on experimental data of creep samples of wood under load and after unloading [13].

Research results of deformation creeping and reverse creeping along fibres allowed to plot rheological wood behaviour functions with taking into account accumulated residual deformations, that are necessary for calculation of stressed-strained lumber state in wood drying process. That's why, when we substitute correlation (15) into formula (16), and then into equation (12), we get equilibrium equations that are similar to Lyame equations, where part of additional volume forces play gradients of temperature and moisture content. So, if to add to the equations (12), (15), (16) and boundary conditions (13), (14) initial condition given by

$$u_i|_{\tau=0} = 0, \quad (20)$$

then we will get nonstationary task for stressed-strained state determination of dried wood.

### Variational task formulation

For the task category to which belongs written above non-stationary task of stressed-strained state determination, is popular formulation on basis on Lagranzh principle (principle of a minimum of full potential energy) (16), that claims the following. Among the acceptable movings  $\mathbf{u}$  of wood as viscous elastic body, which belong to space

$$H_A = \{ \mathbf{u} = (u_1, u_2)^T : u_i|_{x_i=0} = 0, u_i \in W_2^1(\Omega), i = 1, 2 \},$$

are transfers that meet the location of equilibrium and give minimal value to functional of Lagranzh

$$\Pi(u) = \frac{1}{2} \int_{\Omega} \varepsilon^T \mathbf{C} \varepsilon d\Omega + \int_{\Omega} \varepsilon^T \mathbf{C} \int_0^t \mathbf{R}(t, \tau) \varepsilon(\tau) d\tau d\Omega - \int_{\Omega} \varepsilon^T \mathbf{C} (\alpha \Delta T + \beta \Delta U) d\Omega. \quad (21)$$

When to substitute into functional (21) expressions (15), (16), we'll get

$$\Pi(u) = \frac{1}{2} \int_{\Omega} \mathbf{u}^T \mathbf{B}^T \mathbf{C} \mathbf{B} \mathbf{u} d\Omega + \int_{\Omega} \mathbf{u}^T \mathbf{B}^T \mathbf{C} \int_0^t \mathbf{R}(t, \tau) \mathbf{B} \mathbf{u} d\tau d\Omega - \int_{\Omega} \mathbf{u}^T \mathbf{B}^T \mathbf{C} (\alpha \Delta T + \beta \Delta U) d\Omega, \quad (22)$$

Decision of task about minimum of functional (22) with the help of finite element method is searched in finite subspace  $S_N$  of energetic space  $H_A$ . Basic functions are determined on quadrangles, that

cover with grid region  $\Omega$  and intersect between each other. In that case transfers on each element express through nodal values of transfers. So, we have:

$$u_1(\mathbf{x}, \tau) = \sum_{i=1}^N u_{1i}(\tau) \varphi_i(\mathbf{x}); \quad u_2(\mathbf{x}, \tau) = \sum_{i=1}^N u_{2i}(\tau) \varphi_i(\mathbf{x}). \quad (23)$$

Let's input dissection for time using the rule  $t_k = \tau_k = k\Delta\tau$ ,  $\Delta\tau = \frac{\theta}{S}$ , where  $S$  – integer, and mark  $\mathbf{u}_k = \{u_1(\tau_k), u_2(\tau_k)\}^T$ . When to put correlation (23) into functional (22) and sum all finite elements, from minimum conditions of functional (22)  $\delta\Pi = 0$ , then we'll get on each step by time, system of linear algebraic(al) equations as:

$$\begin{aligned} & \frac{1}{2} \int_{\Omega} \mathbf{B}^T \mathbf{C} \mathbf{B} \mathbf{u}_k d\Omega + \frac{\Delta\tau}{2} \int_{\Omega} \mathbf{B}^T \mathbf{C} \mathbf{R}(t_k, \tau_k) \mathbf{B} \mathbf{u}_k d\Omega = \\ & = \int_{\Omega} \mathbf{u}^T \mathbf{B}^T \mathbf{C} (\alpha \Delta T + \beta \Delta U) d\Omega - \sum_{i=1}^{k-1} \Delta\tau \int_{\Omega} \mathbf{B}^T \mathbf{C} \mathbf{R}(t_k, \tau_i) \mathbf{B} \mathbf{u}_i d\Omega. \end{aligned} \quad (24)$$

Correlation (24) makes it clear that calculated on k-step transfer vector components  $u_k$  (in the left part) depend on gradient to temperature and moisture and from previous states of system (in the right part).

### Analysis of the results of numerical modeling

For numerical research of processes of heat-moisture-transfer and deformation of wood used the following parameters of the environment and heat-moisture-exchange: for  $U > 0.35$ , temperature  $t_0 = 79^\circ\text{C}$ ,  $\varphi = 0.77$ , relative humidity of the drying agent. The properties used as reference were obtained from the literature [10, 12] and are given for a wood:  $\rho_0 = 460 \text{ kg/m}^3$ ;  $k_q = 455.6 \text{ J/m}\cdot\text{K}\cdot\text{h}$ ;  $c_q = 1630 \text{ J/kg}\cdot\text{K}$ ;  $\lambda = 2.3 \cdot 10^5 \text{ J/kg}$ ;  $k_p = 4.5 \cdot 10^{-7} \text{ kg}\cdot\text{m/h}\cdot\text{N}$ ;  $c_p = 1.2 \cdot 10^{-3} \text{ m}^2/\text{N}$ ;  $\delta = 0.02 \text{ 1/K}$ ;  $a_m = 6.01 \cdot 10^{-3} \text{ kg/m}^2\cdot\text{h}\cdot\text{M}$ ;  $a_q = 1.5 \cdot 10^5 \text{ J/m}^2\cdot\text{K}\cdot\text{h}$ ;  $c_m = 0.003 \text{ kg/kg}\cdot\text{M}$ ;  $\varepsilon = 0.1$ ;  $k_m = 3.6 \text{ kg/m}\cdot\text{h}\cdot\text{M}$ .

Thermal parameters of wood drying process are determined by the relations of approximation [14, 15] from moisture and temperature. The formulas for their determination presented in [12]. But for some parameters needed clarification. In particular, the coefficients of moisture conductivity of wood according to [1] depends only on the temperature. Research data [15, 16] indicate a significant dependence on moisture. Therefore, for the definition of addiction as a function of used the results of experimental studies [6, 15]. Obtained dependence are as follows:

$$a_m(t) = (a_{t1}t^3 + a_{t2}t^2 + a_{t3}t + a_{t0}) \cdot 10^{-10}, \text{ m/s}, \quad (25)$$

On the basis of experimental data processing [12, 14, 17] obtained the dependence of the coefficient of moisture conductivity of wood from moisture to constant temperature:

$$a_m(U) = -274,391u^5 + 634,908u^4 - 526,7u^3 + 181,864u^2 - 22,655u + 1,905 \quad (26)$$

Then for the calculation was that:

$$a_m = a_{mt} \cdot a_{mu}; \quad a_{m\text{pad}}/a_{m\text{max}} = 1,25.$$

Table 1

### The value of the coefficients for different types of wood

	$a_{t1}$	$a_{t2}$	$a_{t3}$	$a_{t4}$
Pine	$1,273 \cdot 10^{-5}$	$9,74 \cdot 10^{-4}$	0,022	0,587
Hardwood	$8,565 \cdot 10^{-5}$	$-2,361 \cdot 10^{-3}$	0,111	0,192
Oak	$8,565 \cdot 10^{-5}$	$-5,704 \cdot 10^{-4}$	0,041	0,012

For the determination of the coefficient of moisture-Exchange used dependency and nomogram are resulted known [2]. Her analysis reveals about the independence of the coefficient of moisture-Exchange from the wood and moisture of the material, but it depends on the relative moisture of the drying agent. Analysis of nomogram shows the achievement of a coefficient of moisture-exchange of identical values for  $\varphi = 0,1$  and  $\varphi = 0,75$  for air speed of 2 m/s. Available at the different equilibrium moisture content cause a certain ambiguity. Therefore, for the determination of the coefficient of moisture-exchange used the formula [6, 15]:

$$\alpha_m = 0,95 \left( \frac{T}{(P/P_a)\varphi\varepsilon} \right) \cdot 10^{-9}, \quad (27)$$

where  $\varepsilon$  is the criterion of phase transition, T is absolute temperature of environment, K.

It is known [1, 15], that in the capillaries of the wood is reduced the relative pressure of steam. According to the formula of Thomson's pressure of steam in the capillaries is defined by the formula:

$$P = P_0 \exp(-2\sigma V_p / rTR), Pa, \quad (28)$$

where  $\sigma$  is surface tension of liquid, N/m,  $V_p$  is the molar volume of liquid, m<sup>3</sup>/mol.

Then the magnitude of the pressure drop of steam, and therefore the relative moisture  $\varphi$  is:

$$\frac{P}{P_0} = \exp(-2\sigma V_p / rTR). \quad (29)$$

The magnitude of r is given by [18] of the form  $r = r(U)$ , obtained on the basis of the simulation of the structure of the wood system is not permanent capillaries, set the cylinder radius r, which depends on the moisture.

In fig. 1 shows the value depending on the temperature and moisture in the hygroscopic region.

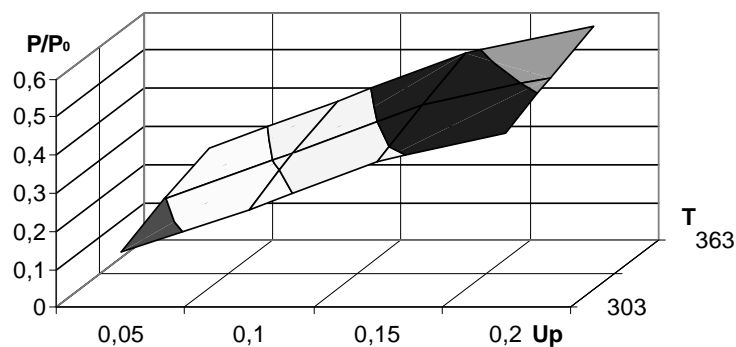


Fig. 1. The dependence of the magnitude of temperature and moisture

The value of  $\varphi$  is obtained by logarithm the magnitude of  $W_p$ . We obtain the following formula:

$$\varphi = 0,42 \ln \frac{100U_p}{7,36 - 0,015T}. \quad (30)$$

Analysis of graphical dependencies distribution moisture content and temperature in wood plate (fig. 2 and fig. 3) indicates that, despite the higher value of the temperature in the solid phase compared with liquid, the intensity achieve uniform values in the process of drying the different phases is higher than in the solid. Such a distribution of values of moisture content, temperature, and velocity change in different phases of conditioned by higher temperature conductivity waters compared with the heat-exchange solid phase. Graphic dependence non uniform distribution of moisture especially due to phase transitions between liquid and vapor-air phases. However, in the initial stages of the transition process of intense change of temperature and moisture for all phases is observed on the surface of the material. It should be noted that the difference in distribution of temperature fields over the duration of the drying of wood increases, namely temperature solid phase increases, increasing the temperature of the liquid phase is slowed down and does not exceed the temperature of saturated steam.

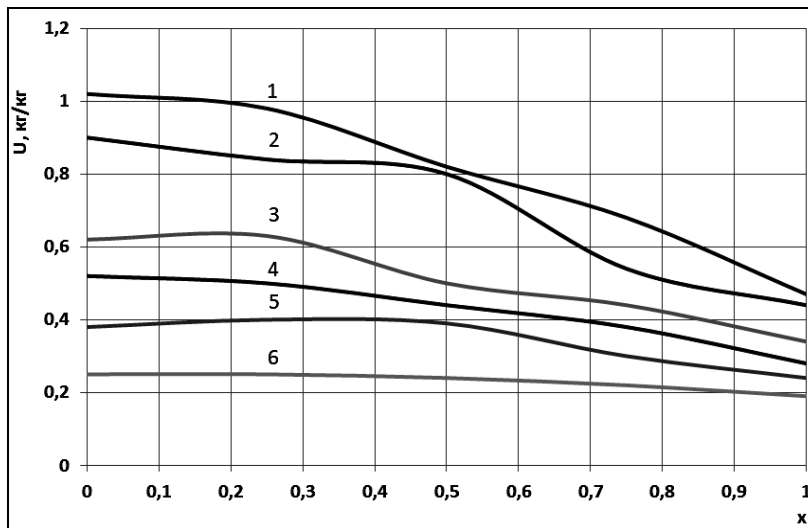


Fig. 2. The distribution of moisture content of solid phase in a wood plate for different values of time (curve 1 – 10 h; 2 – 20 h; 3 – 30 h; 4 – 40 h; 5 – 50 h; 6 – 60 h)

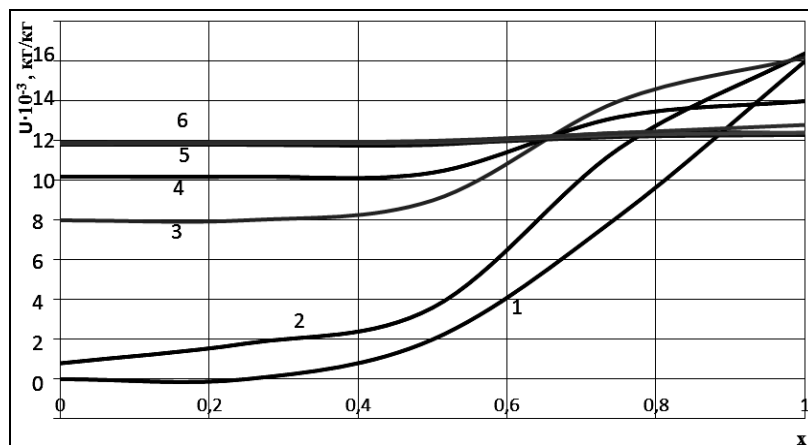


Fig. 3. Distribution of the moisture content of the air-vapor phase in wood plate for different values of time (curve 1 – 10 h; 2 – 20 h; 3 – 30 h; 4 – 40 h; 5 – 50 h; 6 – 60 h)

The temperature in the gas phase reaches values that are higher than in other phases. In addition, the intensity of the changes of vapor-gas and liquid phases have significantly changed in the process of dehydration. There is a significant effect of structural anisotropy of wood on these processes. At the initial stage of the process models for lumber radial direction mentioned vapor-gas mixture increases from the central part of the surface. For tangential samples distribution of steam-gas mixture is more uniform. The intensity of the phase transition for the initial stages of dehydration of wood varies in different points of the wood and essentially depends on the pressure of the gas mixture.

Figure 4 describes the change in the volumetric content of liquid phase in mineral plate, fig. 5 describes the distribution of concentration of air depending on changes in time. The results published in mathematical modeling are consistent with the results of experimental research and data on the distribution of temperature and moisture in a homogeneous environment for particular cases.

In particular, the model temperature solid phase is close to the measured surface temperature and temperature of liquid phase more than meets the temperature at the center of the wooden plate by known experimental data. Original uneven distribution of moisture significantly affects the distribution of moisture in wood as a result of the evaporation of the liquid and the disappearance of the steam phase. In the process of dehydration wood reduction zone forming the maximum values of the pressure of the gas mixture, and values are reduced.



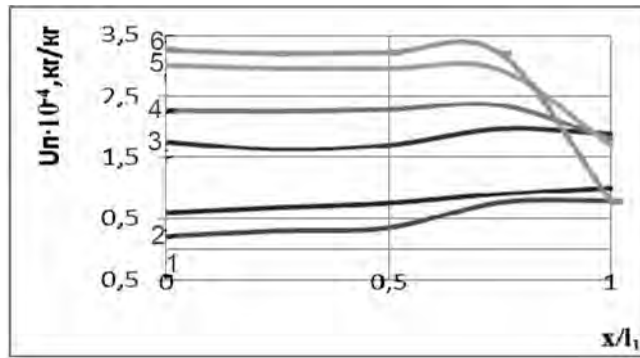


Fig. 4. Distribution of the moisture content of the liquid phase in mineral plate for different values of time (curve 1 – 10 h; 2 – 20 h; 3 – 30 h; 4 – 40 h; 5 – 50 h; 6 – 60 h)

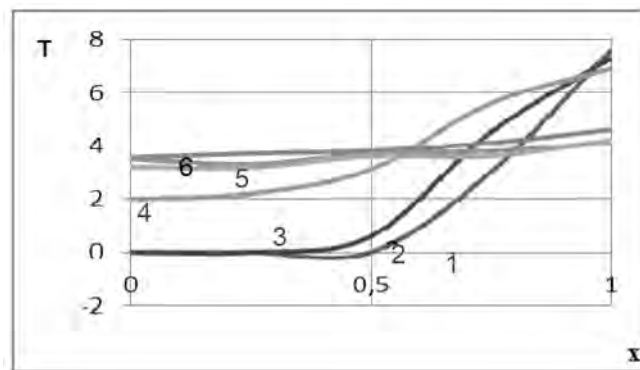


Fig. 5. Distribution of concentration of air in wood plate for different values of time (curve 1 – 10 h; 2 – 20 h; 3 – 30 h; 4 – 40 h; 5 – 50 h; 6 – 60 h)

Analysis of the distribution of temperature and moisture transfer in liquid, solid and air phases indicates that the mathematical models allow to predict the specifics of the interrelated processes of transference in different phases and take into account the physical linearity of these processes, occasioned by dependence of physical properties of the wood from the heat and moisture.

Determining the cause of the wood internal pressure and its gradients is a process of intensive expansion of vapor gas mixture in the material, due to the heating and the heterogeneity of capillary-porous structure. Increase the temperature of the material is intensifying its internal process of vaporization and molar transfer moisture to the surface of the material. Analysis of the drawing dependency indicates that in the process of drying of wood ( $460 \text{ kg/m}^3$ ) with an initial moisture  $\text{kg/kg}$  is characterized by three periods: the period of warming up, a period of constant speed drying and falling speed drying period. For the first period in the wood carried out intensive heating, which causes rapid vaporization. The temperature of the surface layers of the wood during the whole period is greater than in the central layers. In the most heated layers of wood until the end of the first period of drying process occurs the maximum residual pressure. This pressure raises the temperature for the material, which is lower than the saturation temperature for a given atmospheric pressure.

The second period of drying process is characterized by the relative stabilization of growth temperature field. This is caused by the absorption of a significant number of the internal heat of vaporization, which in turn causes rapid growth of residual internal pressure. The presence of this gradient of pressure aimed at the internal layers of wood, intensifying its selection process. At the beginning of a period of constant speed drying there is a combination of the fronts of the maximum values of the internal residual pressure. In the middle of this period, its value reaches the maximum values and then observed their gradual reduction. Increase the values of pressure in wood drying process are determined by the structure of the material over a long time air mixture.

An important aspect is the interaction of temperature and internal pressure (fig. 6, fig. 7). Since the temperature of the wood in the process of drying gradually increases, the value of total internal pressure in the story is greater than the saturated vapour pressure values for the temperature of the material. The pressure of the gas-vapor mixture has the maximum value in the central area of the wood plate. This relationship between the temperature of the wood and the internal pressure is yet another confirmation of the fact that in the process of drying is carried out an intensive internal vaporization or evaporation of moisture.

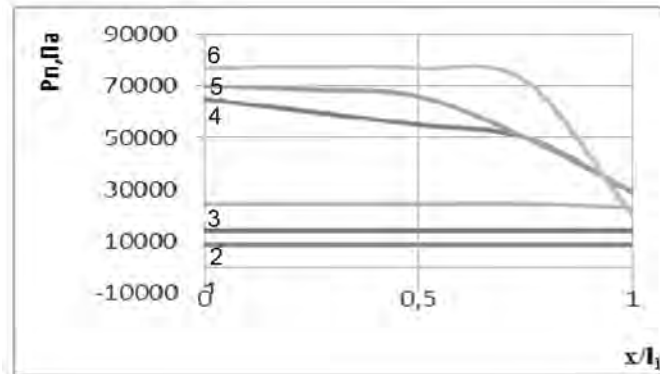


Fig. 6. Distribution of the vapor pressure in wooden plates for different values of time (curve 1 – 10 h; 2 – 20 h; 3 – 30 h; 4 – 40 h; 5 – 50 h; 6 – 60 h)

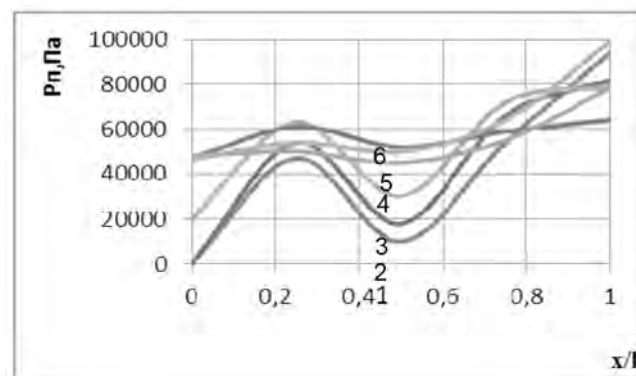


Fig. 7. Distribution of air pressure in the wood plate for different values of time (curve 1 – 10 h; 2 – 20 h; 3 – 30 h; 4 – 40 h; 5 – 50 h; 6 – 60 h)

Analysis of the temperature field for the period flowing speed drying indicates their growth throughout the volume of the material as long as the temperature of the central layer does not come up with the temperature of the surface layers. Because this period is a distinctive selection of bound moisture, then the length of this period is quite significant. However, the intensity of decreasing internal residual pressure is greater in comparison with the previous periods.

Capillary-porous medium wood drying process except the moisture filled and vapor-air mixture. Capillary-porous medium wood drying process except the moisture filled and vapor-air mixture. In the process of heating the wood to a liquid (water), and steam and air is trying to increase its volume. However, changing the volume of wood and liquid phases of the wood with the increase in temperature is negligible compared with the increase in the volume of vapor-air mixture. The increase in temperature and decrease moisture of wood during drying causes the increase of gas permeability in the material. That is, the number of steam-air mixture is changed during the drying process. Since, during the drying process over a long period remains the air, it causes a reduction of pressure in the material, the value of which more the pressure of water vapor at a given temperature.

## Conclusions

Resolved an important for the intensive process of drying the task of diffusion-filtration-products-heat-transfer in which taken into account anisotropy of temperature, humidity and filtration characteristics of material. Formulated a mathematical model of the viscous-elastic deformation of wood, which takes into account the accumulation of irreversible deformation and anisotropy variables of heat mechanical characteristics of wood. Conducted a numerical analysis of the influence of technological process parameters are highly intensive drying of wood on two dimensional temperature-humidity and viscous-elastic material status.

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