Mathematical Model Of Stress And Deformation State Of Glued Anchorage Of Reinforcement Bars In Cases Of Exposure Short And Long Term Loading

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Abstract. The solution of axisymmetric task of the theory of elasticity for the case of embedment of reinforcement bars into concrete of crescent type by acrylic glues, allowing to determine tensions and deformations both on contacts glue-anchor, glue-concrete and in the concrete body has been proposed. It has been taken into account that at long-term loading the behaviour of acrylic glues co-oridinates the linear theory of creep. The results of calculation experiment are presented.

Key words: reinforcement bar, crescent profile, concrete, acrylic glue, tension.

I. Introduction

In conditions of reconstruction of industrial buildings and structures the necessity of reliable fastening of structures in the shortest terms without stopping the production processes appears. Such structural fastenings can be carried out using polymeric glues including acrylic glues.

As reinforced steel of crescent profile is used in construction in Ukraine according to the state standards experimental investigation of the strength and deformation of reinforcement bars of this profile embedded into concrete using acrylic glues of different compositions has been carried out at O.M. Beketov National University of Urban Economy in Kharkiv. The urgency of this investigation was caused by the results of comparative analytical studies of geometric characteristics of reinforcement bars of class A-III and crescent profile of A500C class. Module values of ribs pitch of reinforcement bars have been obtained.

The comparison of geometric parameters of ring and crescent reinforcement rolled stock shows that the area of fastening to reinforcement concrete of A-III class is much larger than of reinforcement class A500C. Besides, they have different values of temporal resistance and flow limit.

In connection with that experimental investigations on determining the strength of reinforcement bars of class A500C anchorage into concrete using acrylic glues of different compositions have been carried out at O.M. Beketov National University of Urban Economy in Kharkiv. The experiments [1-2] showed that the strength of the embedment of reinforcement bars of class A500C into concrete using acrylic glues of ordinary compositions is provided at \(l_{anch} = 22,5d_r\).

The use of special adds increased the strength characteristics of acrylic glue [3] and allowed to reduce the depth of embedment of mentioned above bars to \(17,5d_r\) (\(d_r\) – diameter of reinforcement bars) [1].

II. The Solution of the Task of Stress and Deformation State of Glued Anchorage

The investigation of long-term and short-term strength of mentioned anchorage showed their sufficient reliability. The use of such anchor fastenings required studying their stress state at short-term and long-term influence of out-pulling efforts on the reinforcement bar (Fig.1).

The results of the investigations testify that at long-term load the behavior of acrylic glues conforms to the linear theory of creeping [3]. When considering the stress state of rotation bodies under the influence of axisymmetric stress load and the shift in conditions of linear creeping [4] can be expressed through bi-harmonic function of A. Ljav [5]:

\[
\sigma_z = \frac{\partial}{\partial z} \left\{ (2 - \nu) \Delta^{2} \Phi -\frac{\partial^2 \Phi}{\partial z^2} \right\};
\]

\[
\sigma_r = \frac{\partial}{\partial r} \left\{ \nu \Delta^{2} \Phi - \frac{\partial^2 \Phi}{\partial r^2} \right\};
\]

\[
\sigma_{\theta} = \frac{\partial}{\partial \theta} \left\{ \nu \Delta^{2} \Phi - \frac{1}{r} \frac{\partial \Phi}{\partial r} \right\};
\]

\[
\tau_{rz} = \frac{\partial}{\partial z} \left\{ (1 - \nu) \Delta^{2} \Phi -\frac{\partial^2 \Phi}{\partial z^2} \right\};
\]

\[
u = 1 + \nu \frac{\partial^2 \Phi}{\partial r \partial z \partial \tau} + \int_{\tau_1}^{\tau} (1 + \nu) \Delta^{2} \Phi \frac{\partial \omega(t, \tau)}{\partial z} + \frac{\partial}{\partial z} \omega(t, \tau) d \tau; 
\]

\[
\omega = 1 + \nu \left\{ 2(1 - \nu) \Delta^{2} \Phi \frac{\partial^2 \Phi}{\partial z^2} \right\} + 
\int_{\tau_1}^{\tau} \left\{ 2(1 - \nu) \frac{\partial^2 \Phi}{\partial z^2} \right\} \frac{\partial \omega(t, \tau)}{\partial t} + \omega(t, \tau) d \tau;
\]

\[
\Delta^4 \Phi(t, r, z) = 0;
\]

\[
\omega(t, \tau) = \frac{1}{E(\tau)} + G(t, \tau).
\]

where \(\nu\) – the coefficient of Poisson; \(r, z\) – cylindrical coordinates; \(\tau\) – the age of acrylic glue; \(t\) – the moment of time, for which stress state is determined; \(\tau_1\) – the age of acrylic glue, for which stress state is determined; \(E(\tau)\) – momentary modulus of acrylic glue elasticity; \(G\) – the measure of creeping.

Substitute the stress function in the following way:

\[
\Phi(t, r, z) = \varphi(t) \chi(r, z).
\]

According to [5] obtain the following expression:

\[
\Phi(t, r, z) = \left[ D(t) \sin mz + D(t) z \cos mz \right] K_0(mv).
\]

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From boundary conditions at the ends of anchor fastening:

\[
[\sigma_{rz}]_{z=1} = [r_{rz}]_{z=0} = 0. \tag{11}
\]

According to expressions (1) and (4) find the functions \( A(t) \) and \( D(t) \) connected with the relationship:

\[ mA(t) = -D(t)((2\nu-1) - \lambda t g \lambda], \tag{12} \]

where: \( m = \frac{\lambda}{l}; \lambda \) – the root of transcendental equation.

\[
sin \lambda \cos \lambda + \lambda = 0. \tag{13} \]

\( l \) – the depth of reinforcement bar embedment into concrete.

Transcendental equation (13) has infinite number of roots \( \lambda_s = \xi_s + i\eta_s; s = 1, 2, 3, \ldots; m_s = \lambda_s / l. \)

That is why the stress function, corresponding to the boundary conditions (11), may be written in the following way:

\[
\sum_s D_s(t) \left[ z \cos \frac{\lambda_s z}{l} - \frac{1}{\lambda_s} \left( 2\nu - 1 - \lambda_s t g \lambda_s \right) \right] K_0 \left( \frac{\lambda_s r}{l} \right). \tag{14}
\]

Taking into account the solutions obtained before [4, 5], substituting the equation (14) into (1-6) obtain the equation for the stresses and shifts in acrylic glue:

\[
\tau_{rz} = -\sum_s D_s(t) \left[ \cos^2 \frac{\lambda_s z}{l} \sin \frac{\lambda_s z}{l} + \frac{\lambda_s z}{l} \sin \frac{\lambda_s z}{l} K_0 \left( \frac{\lambda_s r}{l} \right) \right]; \tag{15}
\]

\[
\sigma_z = -\sum_s D_s(t) \left[ \sin^2 \frac{\lambda_s z}{l} \cos \frac{\lambda_s z}{l} + \frac{\lambda_s z}{l} \cos \frac{\lambda_s z}{l} K_0 \left( \frac{\lambda_s r}{l} \right) \right] + \sigma + \frac{\lambda_s z}{l} \sin \frac{\lambda_s z}{l} K_0 \left( \frac{\lambda_s r}{l} \right); \tag{16}
\]

\[
\sigma_r = \sum_s D_s(t) \left[ \frac{\lambda_s z}{l} \sin \frac{\lambda_s z}{l} \left( 1 + \cos^2 \lambda_s \right) \right] \times \cos \frac{\lambda_s z}{l} K_0 \left( \frac{\lambda_s r}{l} \right) + \frac{1}{r} \frac{\lambda_s z}{l} \sin \frac{\lambda_s z}{l} K_0 \left( \frac{\lambda_s r}{l} \right) - \left( 1 + \cos^2 \lambda_s - 2\nu \right) \cos \frac{\lambda_s z}{l} K_0 \left( \frac{\lambda_s r}{l} \right); \tag{17}
\]

\[
u = \frac{1 + \nu}{E(t)} \sum_s D_s(t) \phi_s(r, z) - \sum_s \phi_s(r, z)(1 + \nu) \times \left( \int \frac{D_s(t)}{\partial \tau} \omega(t, \tau) d\tau \right) ; \tag{18}
\]

\[
\omega = \frac{1 + \nu}{E(t)} \sum_s D_s(t) \psi_s(r, z) - \sum_s \psi_s(r, z)(1 + \nu) \times \left( \int \frac{D_s(t)}{\partial \tau} \omega(t, \tau) d\tau \right) ; \tag{19}
\]

where: \( z \) – the coordinate of the point, from which the stress or shifts are determined; \( K_0 \) – the function of Mc Donald;

Consider the boundary conditions on the surface of the contact of anchor with acrylic glue in case of its pulling out, from which the relation for determining the function \( D(t) \) can be obtained. Boundary conditions include the continuity of tangent and radial stresses, relative axis extensions and radial shifts on the contact glue – anchor as well as the equality \( \sigma_z = 0 \) at \( z = 0 \).

Starting with some age \( \tau_1 > \tau \) (\( \tau = 10 \ldots 25 \) days), the increment of glue elasticity modulus stops, so the bellowing equality can be used:

\[
n(t) = \frac{E_S}{E_t} \approx \frac{E_S}{E_{(o)}} = \frac{E_S}{E_K} = n_K. \tag{20}
\]

In accordance with [4, 5] accept the following expressions for creeping measure:

\[
G(t, \tau) = \phi(t) \left[ 1 - e^{-\gamma(t-\tau)} \right], \tag{21}
\]

\[
\phi(t) = C_o + A_1; \tag{22}
\]

\[
R(t, \tau) = \frac{1}{E_o \sigma} \left[ 1 - e^{-\gamma(t-\tau)} \right] E \tag{23}
\]

In these expressions \( C_o, A_1, \gamma \) – the constants, depending on the composition and strength of acrylic glue and determined by experiments. The structure of the core in the expression (23) can be simplified, if to consider the process of acrylic glue creeping at a mature age.

Analytical dependence of creeping measure [1] fits to the experimental data for acrylic glue at the age of more than 25 days:

\[
G(t - \tau) = \sum_{k=1}^{m} A_k l^{-\gamma_k(t-\tau)}; \tag{24}
\]

\[
A_k \geq 0; \gamma_o = 0; \gamma_k \geq 0;
\]

\[
R(t - \tau) = -\sum_{k=0}^{m} A_k y e^{-\gamma_k(t-\tau)} E; \tag{25}
\]

at \( k = 1 \) \( R(t - \tau) = -A_1 y e^{-\gamma_1(t-\tau)} E. \tag{26}\)
Fig. 2. Distribution of tangent (a), normal axis (b), radial (c) and hoop (d) stresses in the layer of acrylic glue on the contacts glue-anchor (1) and glue-concrete (2).

III. Calculation Experiment

Taking into account boundary conditions of expressions (21 – 24) as well as the start of loading time reading $t = 0$ determine the stress function $D_i(t)$. Substitute the values of $D_i(t)$ into formulas (15-17) and obtain the stress values $\sigma_z^{(k)}$, $\sigma_r^{(k)}$, $\sigma_\theta^{(k)}$ in case of maximum stress state of anchor fastening when pulling out the bar embedded into concrete using acrylic glue.

The graphs of distribution of tangent, normal axis, radial and hoop stresses in the layer of acrylic glue on the contacts glue-anchor and glue-concrete are given in fig. 2.

The curves of stress distribution calculated for the case of elastic-momentary use of pulling out effort are marked by solid lines and the curves of stress distribution as a...
result of acrylic glue creeping during 100 days are marked by dotted lines.

The graphs of distribution of the axial and radial displacements in the layer of acrylic glue on the contacts glue-anchor and glue-concrete are given in fig.3.

Calculation is made for anchor fastening with materials having the following characteristics: anchor elasticity modules \( E_S = 2 \times 10^5 \) MPa, glue \( E_k = 8,78 \times 10^3 \) MPa, concrete \( E_b = 2,3 \times 10^4 \) MPa, the coefficients of Poisson \( \nu_S = 0,25, \nu_k = 0,35 \) and \( \nu_b = 0,16 \). Geometric characteristics of anchor fastening: embedment depth \( l = 17,5d_s \); anchor diameter \( d_s = 2,0\) cm; hole diameter \( d_h = 4,0\) cm; \( l_0 = 20; l_1 = 10 \).

Conclusion

Thus, the solution presented allows determining the stress and deformations both on contacts glue-anchor, glue-concrete and in the concrete body.

References


