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ABOUT EVALUATION OF THE COMPLEX CONTROL SYSTEMS

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Methods for local and general evaluation of the complex control systems are proposed. The set of other problems which has been solved by means of given methods are considered. Principles of parallel organization of calculations based on these methods are described.

Keywords: complex systems, control, optimality, evaluation, parallel algorithms.

1. Introduction

Under mathematical investigation of real complex systems arise a whole series of problems: extraction, processing, and analysis of information about system's motion, creation of models which adequately reconstruct the lows of this motion, computer modeling of the processes proceeding in such systems etc [1]. This is realized both for fundamental learn their laws of motion and availability to use in the case of designing and optimization such systems. Evaluation of quality and reliability and prognostication of behavior of the complex control systems are very important too.

Data about system's functioning are received continuously and their analysis and processing are need. Such data level is too large in order to orient in real time with the purpose of evaluation the state of investigated system. Corresponding generalization is need so development of the methods for solution formulated problem. The set of corresponding problems includes multi parameter and multi criterion evaluation of behavior of separate system's elements, multilevel evaluation their quality including estimation of system's functioning as a whole [2], definition of system's reaction on various internal and external influences [3], prognostication their behavior under functioning in usual and extreme modes. At the same time the problem of definition of the system's sensitivity to various evaluative parameters and criterions [3], problem of choice of optimal mode their functioning [4], problem of choice an optimal system from the class of equivalent systems [5] are arising. Efficient processing and analysis such data level are need. Therefore we must use modern computer technologies especially the methods of parallel calculations [6].

2. Formulation of Problem

Let $A(t) = \{A_j(t)\}_{j=1}^{m}$, $t \in [0,T]$, is the set of characters describing functioning given dynamical control system, T is an interval of test investigations. Every character $A_j(t) = \{A_j^i(t)\}_{i=1}^{n}$, $j=1,2,\ldots m$, is constant vector or vector-function describing separate aspects of system's functioning (initial, intermediate and final states, phase and control vectors etc). We can obtain characters of system's motion both using experimental investigations and by means of mathematical modeling.

Assume that given system function in l modes (or realize l functions) R_k, $k=1,2,\ldots,l$, and for evaluation of quality use s criterions K_p, $p=1,2,\ldots,s$. Limits of indexes and t we will omit below. Denote $\Omega_{ji}^{kp}(t)$ an area of permissible values for component $A_j^i(t)$ for p evaluative criterion when the system functions in k mode.

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Systems of the same type and assignation we will be considered to equivalent if their law of motion describes by the set of characters $A(t)$. Denote G_N class of equivalent systems included *N* elements. Evaluation of quality we will carry out within the limits of G_N by means of comparison the systems belong to given class.

Assume that given system is a new element of the class G_N. Formulate the next problem: 1) on the base of numerical analysis of characters $\{A_j(t)\}_{j=1}^{m}$ to form the flow of local evaluations for components of characters $\{A_j^i(t)\}_{j=1,\ i=1}^{m,\ \ n}$ by the set of criterions K_p, *p=1,2,…,s*, on the given set of function modes R_k, *k=1,2,…,l*; 2) based on obtained set of the local evaluations to form the flow of suspended averaged evaluations of the various level of generalization for established set of parameters, criterions, and modes right up to final conclusion about quality of the system as a whole.

Forming of the set of criterions and parameters is the important problem for solution of above mentioned problems. We can use common criterion functions which suit for investigation of arbitrary systems [7, 8] or specific which are defined by features of given class of systems or the purpose of investigations [9, 10].

Assume that carry out an evaluation of the system by criterion *p* if it is functioning in mode *k*. Introduce auxiliary function $\alpha_j^i(t)$ such that $\alpha_j^i(t)=0$ if $A_j^i(t)\in\Omega_{ji}^{kp}(t)$, $\alpha_j^i(t)=A_j^i(t)-\max\Omega_{ji}^{kp}(t)$ if $A_j^i(t)>\Omega_{ji}^{kp}(t)$, and $\alpha_j^i(t)=\min\Omega_{ji}^{kp}(t)-A_j^i(t)$ if $A_j^i(t)<\Omega_{ji}^{kp}(t)$. As numerical parameters of local evaluations for component *i* of character *j* by criterion *p* in mode *k* we use the values

$$c(A_j^i,K_p,R_k)=\left\|\alpha_j^i\right\|_{C[0,T]},\ l(A_j^i,K_p,R_k)=\left\|\alpha_j^i\right\|_{L_2[0,T]}.$$

3. Local Evaluation of the Complex Systems

The number of numerical parameters of the local evaluations $S=2mnls$ is sufficiently great. Therefore their direct analysis is the complex problem.

Let for characters $\{A_j^i(t)\}_{j=1}^{m}$ of the systems appurtenant to the class G_N are defined values

$$c_{\min}(A_j^i,K_p,R_k)=\min_{G_N}c(A_j^i,K_p,R_k),\ c_{\max}(A_j^i,K_p,R_k)=\max_{G_N}c(A_j^i,K_p,R_k),$$

$$l_{\min}(A_j^i,K_p,R_k)=\min_{G_N}l(A_j^i,K_p,R_k),\ l_{\max}(A_j^i,K_p,R_k)=\max_{G_N}l(A_j^i,K_p,R_k).$$

Introduce for every function $A_j^i(t)$ of given control system integer-valued arrays of local evaluations by means of formulas

$$C_{ji}^{kp}=[(c(A_j^i,K_p,R_k)-c_{\min}(A_j^i,K_p,R_k))/(c_{\max}(A_j^i,K_p,R_k)-c_{\min}(A_j^i,K_p,R_k))\times I]+1,$$

$$L_{ji}^{kp}=[(l(A_j^i,K_p,R_k)-l_{\min}(A_j^i,K_p,R_k))/(l_{\max}(A_j^i,K_p,R_k)-l_{\min}(A_j^i,K_p,R_k))\times I]+1.$$

Here $I\geq 2$ is a number of local evaluations and [*x*] is an aliquot of *x*.

Usually two or three levels of evaluations are enough but for prognostication of system's behavior by means of extrapolation of evaluation in time the value *I* are need to take greater.

By means of local evaluations we can obtain some conclusions about behavior of investigated control system in particular trace their elements functioning unsatisfactorily. But ambiguity and antipathy such conclusions are obvious since the same quantity of equal evaluations don't mean the same quality corresponding systems as a whole.

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4. Generalized Evaluation of the Complex Dynamical Systems

On the base of the set of local evaluations form the succession of suspended averaged evaluations of the various level of generalization right up to final conclusion about quality of the system as a whole. This forming we will carry out in two directions which conditionally denote as vertical and horizontal.

Evaluation in vertical direction allows to analyse the behavior of separate system's characters or the system as a whole by corresponding parameter, criterion, or mode:

1) For separate type of evaluative parameter by the set of every character's component for the each evaluative criterion

$$V^{k,p}_{j,C} = \sum_{i=1}^{n} \rho_{A^i_j} C^{kp}_{ji} \Big/ \sum_{i=1}^{n} \rho_{A^i_j}, \quad V^{k,p}_{j,L} = \sum_{i=1}^{n} \rho_{A^i_j} L^{kp}_{ji} \Big/ \sum_{i=1}^{n} \rho_{A^i_j}.$$

Here $\{\rho_{A^i_j}\}_{i=1}^{n}$ are the weight coefficients determined the meaning of character's components. Here and later the evaluation by values $V^{k,p}_{j,C}$ ($V^{k,p}_{j,L}$) carry out similarly to evaluations by values C^{kp}_{ji} (L^{kp}_{ji}). The values of weight coefficients are defined by experts which investigate the system depend upon their specificity or the purpose of investigation.

2). For separate evaluative criterion by the set of evaluative parameters

$$V^{k,p}_{j} = (\rho_C V^{k,p}_{j,C} + \rho_L V^{k,p}_{j,L})/(\rho_C + \rho_L).$$

Here ρ_C, ρ_L are the weight coefficients determined the meaning of evaluative parameters.

3). For the separate function mode by the set of evaluative criterions

$$V^{k}_{j} = \sum_{p=1}^{s} \rho_{K_p} V^{k,p}_{j} \Big/ \sum_{p=1}^{s} \rho_{K_p}.$$

Here $\{\rho_{K_p}\}_{p=1}^{s}$ are the weight coefficients determined the meaning of evaluative criterions.

Evaluation in horizontal direction allows to analyse the behavior of separate components of concrete character or system's characters as a whole by corresponding parameter, criterion, or mode:

1). For separate type of evaluative parameter on the set of criterions for every component of character

$$H^{j,k}_{i,C} = \sum_{p=1}^{s} \rho_{K_p} C^{kp}_{ji} \Big/ \sum_{p=1}^{s} \rho_{K_p}, \quad H^{j,k}_{i,L} = \sum_{p=1}^{s} \rho_{K_p} L^{kp}_{ji} \Big/ \sum_{p=1}^{s} \rho_{K_p}$$

2). For introduced evaluative parameters on the set of criterions for every component of character

$$H^{j,k}_{i} = (\rho_C H^{j,k}_{i,C} + \rho_L H^{j,k}_{i,L})/(\rho_C + \rho_L)$$

3). For separate functioning mode on the set of components of character

$$H^{k}_{j} = \sum_{i=1}^{n} \rho_{A^i_j} H^{j,k}_{i} \Big/ \sum_{i=1}^{n} \rho_{A^i_j}$$

Then we have [2]

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Theorem 1. $H_j^k = V_j^k \equiv VH_j^k$.

Evaluations of the last level define behavior of separate system's characters by the set of criterions for selected modes of their functioning. On the base of these evaluations we can define optimal mode of functioning for given system [4].

The next step to obtain the suspended averaged evaluations defines in the following way. In vertical direction it consists in evaluation by the set of characters for every function mode

$$VH^k = \sum_{j=1}^{m} \rho_{A_j} VH_j^k \Big/ \sum_{j=1}^{m} \rho_{A_j} .$$

Here $\{\rho_{A_j}\}_{j=1}^{m}$ are the weight coefficients determined the meaning of system's characters.

In horizontal direction it consists in evaluation by the set of modes for every character of investigated system

$$VH_j = \sum_{k=1}^{l} \rho_{R_k} VH_j^k \Big/ \sum_{k=1}^{l} \rho_{R_k} .$$

Here $\{\rho_{R_k}\}_{k=1}^{l}$ are the weight coefficients determined the meaning of function modes. Then we have [2]

Theorem 2. $\sum_{k=1}^{l} \rho_{R_k} VH^k \Big/ \sum_{k=1}^{l} \rho_{R_k} = \sum_{j=1}^{m} \rho_{A_j} VH_j \Big/ \sum_{j=1}^{m} \rho_{A_j} \equiv VH$

Value *VH* allows to take final conclusion about function quality of investigated system as a whole. On the base of last evaluations we can define optimal system from the class G_N equivalent systems [5]. Program FESTIWAL (Fast ESTImation of WALking) [11] is an example of using proposed methods for the evaluation of prosthesis quality of disabled persons.

5. Evaluation of the Complex Systems on the Base of Parallel Calculations

Depend upon solved problem of evaluation into the base of parallel organization of calculations on the first stage use one from approaches:

1) presentation of initial problem in the form of unconnected and data-independent sub problems with approximately equal level of executed calculations;
2) presentation of main fragment of the problem on the form of unconnected sub problems whose solutions are used for final evaluation of function quality of investigated system (as a whole or their character or character's component) i.e. getting of solution for the given problem.

The next stage of parallel organization of calculations consists in interpretation every sub problem as separate problem of evaluation and using for their solving one of above mentioned approaches. This process we continue to level when every sub problem is the problem of local evaluation (see section III).

On the base of considered approaches to parallel organization of calculations we present the next algorithmic constructions.

Construction I:

$$fork(h_1; h_2; ...; h_p)\, join$$

specifies calculations in the form of *p* parallel independent chains [6] h_i, $i=1,2,\ldots p$, for every of which we solve separate sub problem.

Construction II:

$$fork(\tilde{h}_1; \tilde{h}_2; ...; \tilde{h}_{p_0})\, join, \tilde{h}$$

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specifies calculations in the main fragment of given problem in the form of p_0 parallel independent chains $\tilde{h}_j$, $j=1,2,\ldots,p_0$. Their results are using in other fragment $\tilde{h}$. The next acceleration of calculations we can obtain by means of their parallel organization during solving every sub problem. Therefore the next construction is more effective.

Construction III:

$$fork(\ fork(h_1^1;h_1^2;...;h_1^w)\ join, h_1^0;\ \ fork(h_2^1;h_2^2;...;h_2^w)\ join, h_2^0;\ldots;$$
$$fork(h_p^1;h_p^2;...;h_p^w)\ join, h_p^0)\ \ join.$$

It defines calculations in the main fragment for problem k in the form of w parallel independent chains h_k^l, $l=1,2,\ldots,w$.

We can propose other algorithmic structures which take into account some peculiarities of the solving problems. Recommendations about realization of proposed constructions on parallel computing systems with common and distributed memory are considered.

6. Conclusion

On the base of common approach some problems of evaluation of complex control systems are considered. Using scalene, multi-criterion, and multilevel analysis of their main characters we can obtain the evaluations of function quality for the various kinds of control systems, choose optimal modes their functioning, choose optimal system from some class of equivalent systems etc. We can use proposed methods for successive modernization of investigated systems by means of improvement of elements which functioning are unsatisfactorily, comparative evaluation of new and existent system's models etc. At the same time the solutions of some problems we can use for solving other problems. Use of parallel organization of calculations allows to evaluate the function quantity for the systems of arbitrary complexity (technical, biological, economical, social ets.).

References

[1] V.M.Kaziev, "Introduction in analysis, syntesis, and modeling of systems", Intuit.ru, Moscow, 2006.

[2] A.D.Polishchuk, "An optimization of estimation of the functioning quality of the complex dynamical systems", *Jorn.of Automation and Information Sciences*, vol.4, pp.39-44., 2004.

[3] A.D.Polishchuk, "Functioning stability level as quality criterion of the complex dynamical systems", *Proc. Intern. Conf. "Dynamical System Modeling and Stability Investigation"* (DSMSI-2005), p.210, Kiev, May 2005.

[4] A.D.Polishchuk, "Choice of optimal regimes for functioning of complex dynamic systems", *Math.Meth. and Physicomech.fields,* , vol.48, pp.62-67, 2005.

[5] A.D.Polishchuk, "About the choice of the optimal dynamical system from the class of equivalent systems", *Information Extraction and* Processing, vol.20(96), pp23-28, 2004.

[6] V.A.Valkovsky, "Parallelization of algorithms and programs", Radio and Tie, Moscow, 1989.

[7] R.P.Fedorenko, "Numerical solution of the optimal control problems", Nauka, Moscow, 1978.

[8] A.D.Polishchuk, "Estimation of pathology level of the MSS as optimal control problem", *Proc. VI Intern.Conf. "Math. Probl. of mechanics of nonhomogeneous structures*", pp.435-436, Lviv, May 2003

[9] Winter D.A. "The biomechanics and motor control of human gait: normal, elderly and pathological", Univ. of Waterloo Press, Canada, 1991.

[10] S.A.Timashev, "The main problems and methods for evaluation, optimization and reliability control of GPS", *Calculation and reliability control of the great mechanical systems*, pp.30-50, USC AS USSR, Sverdlovsk, 1984.

[11] A.D.Polishchuk, "Optimization of estimation of the MSS", *Computer Math.and Calc. Optimization*, vol.2, pp.360-367, 2001.

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