Statement of the problem

A quantitative analysis of the dynamics of manufacturing systems development should be considered an important direction of economic research. In the simplest case the growth rates of income, for example, are determined as the product of the share of savings and the efficiency of investment costs. However, this approach is quite general, and does not take into account a number of important factors that characterize disposal and renewal of fixed assets, creation of accumulation resources, etc. Specific problems in this area arise at the level of individual production systems, as well as in the process of making investment decisions. It is necessary to account for the peculiarities of fixed assets reproduction paying attention to their structure, the sources of reproduction, as well as the details and parameters of the investment projects.

Analysis of recent research and publications

Theoretical works, mainly on the macro-level, study various aspects of economic growth, innovation development, making investment decisions, etc. An important for practical use is the method of investment projects evaluation, which is based on determination of the so-called Net Present Value. However, this method solves the problem only partially, because it does not take into account a number of important factors that influence the real activity of enterprises.
The formulation of objectives

This paper aims to obtain scientific findings and applicable recommendations based on determination of quantitative dependencies in the relationship of the economic development rates with a number of technical and economic parameters. We are planning to bring the research to the level of numerical computations and practical application.

Presentation of main materials

The study of the dynamics of fixed assets requires such a scheme of their expanded reproduction, which would use the length of service, disposal and introduction of fixed assets, the change in the volume of resources that are allocated to renewal and increase the fixed assets.

To take into account the mentioned characteristics of the development process, we propose the following scheme of interpretations of fixed assets movement when their operating life is T years [1]:

<table>
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<tr>
<th>1st year</th>
<th>2nd year</th>
<th>3rd year</th>
<th>4th year</th>
<th>...</th>
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<tbody>
<tr>
<td>( F_0 )</td>
<td>( F_T )</td>
<td>( F_T )</td>
<td>( F_T )</td>
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<tr>
<td>( F_1 )</td>
<td>( F_{t+1} )</td>
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<td>( F_2 )</td>
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<td>...</td>
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In the scheme, an arrow shows changes in fixed assets. For example, in the first year of the systems’ operation \( F_0 \) represents the cost of disposed assets that are replaced by the new assets of the cost \( F_T \).

It is reasonable to assume that fixed assets are generally acquired at the cost of the financial resources formed from profit and depreciation. As a rule, accumulated is a part of income (\( \alpha \)) and amortization, calculated based on the relevant norms (aF).

By taking into account the cost of fixed assets in the first year and making use of their rate of return (\( r \)), we determine the cost of new assets with balance equation:

\[
F_T = (\alpha \cdot r + a_F)(F_T + F_{t+1} + ... + F_{T-1}).
\]

In the future, we will mainly analyze the dynamics of fixed assets with constant growth rates (\( \eta \)). For such a process the following quantitative relationship between the costs of fixed assets takes place:

\[
F_{i+1} = \eta \cdot F_i, \text{ or } F_i = \eta^i \cdot F_0.
\]

Taking into account the given quantitative relationships between the various assets, equation (1) is equivalent to the following:

\[
\eta^T = (\alpha \cdot r + a_F) \cdot (1 + \eta + ... + \eta^{T-1}).
\]

If we assume the rates to be unknown variables, then they are determined by the algebraic equation of the Tth degree. It can be solved by any method, including iteration.

As the first approximation we can take a value equalto \( \eta_0 = 1 + \alpha \cdot r \). The left and the right sides of the equation are calculated separately, after which the difference of the obtained values is found. In further iterations it is possible to increase the value of the approximation until the difference of both sides of the equation changes the sign to the opposite. To simplify the numerical calculations the equation can be modified in the following way using the sum of geometric progression terms:

\[
\eta^T = (\alpha \cdot r + a_F) \cdot \frac{\eta^T - 1}{\eta - 1}.
\]

As an example we will take dynamics with the following output parameters:

\( T = 10; aF = 0.1; \alpha = 0.5; r = 0.1. \)
Accordingly, the equation for determining the rates is as follows:

\[ \eta^{10} = 0.15 \times \frac{\eta^{10 - 1}}{\eta - 1} \]

Below are the growth rates calculated in accordance with iterative procedure determining the difference of the left and the right sides of the equation:

1. \( \eta_1 = 1.05; \Delta_1 = 1.6289 - 1.8867 = -0.2578 \)
2. \( \eta_2 = 1.06; \Delta_2 = 1.7908 - 1.9771 = -0.1863 \)
3. \( \eta_3 = 1.07; \Delta_3 = 1.9671 - 2.0725 = -0.1054 \)
4. \( \eta_4 = 1.08; \Delta_4 = 2.1589 - 2.1730 = -0.0141 \)
5. \( \eta_5 = 1.09; \Delta_5 = 2.3674 - 2.2789 = 0.0885 \)

Iterative procedure is stopped, as the difference of the left and the right sides of the equation has changed sign from minus to plus. With an absolute accuracy of up to 1 percent of growth rates it can be considered that the growth rates equal \( \eta = 1.08 \), i.e. the growth rates of fixed assets are 8% per annum. In this case the difference analyzed reaches the smallest absolute value.

The equation for the rates (3) makes it possible to justify the procedure of discounting the resource flows that is used in evaluating the effectiveness of one-time investments (projects). For this purpose, the left and the right sides of the equation should be divided by a variable \( \eta \) and then multiplied by the total investments \( K \). Thus we obtain the following relationship:

\[ K = \frac{R}{\eta^1} + \frac{R}{\eta^2} + \ldots + \frac{R}{\eta^t}, \]  

where \( R = \alpha * r * K + a_f * K \).

In this record the annual cash flows generated from profits and depreciation are discounted according to the value of the investment project.

In the economic literature the evaluation of investment projects is carried out by taking the difference between discounted cash flows and investments (NPV). The project is acceptable if the NPV is positive [2, 3].

As for growth rates it is worthwhile to consider three options for evaluation:

1. \( \text{NPV} = 0 \) – the investment project ensures current growth rates.
2. \( \text{NPV} > 0 \) – the project causes increase of growth rates.
3. \( \text{NPV} < 0 \) – the project leads to decline of growth rates.

An important direction of economic dynamics analysis is considering, apart from fixed assets, the company’s working capital. The latter, having been entered into the system once, are used for almost unlimited time.

As an analysis of resource flows shows the equation for growth rates takes in this case the following form:

\[ \eta^T = (\alpha * r + a_f) * (1 + \eta + \ldots + \eta^{T-1}) + \frac{\lambda}{1+\lambda}, \]  

where \( \lambda = \frac{F_O6}{F_{OCн}} \); \( F_O6 \) is the cost of working capital, \( F_{OCн} \) is the fixed assets cost.

For certain investments the equation \( (K = F_{OCн} + F_O6) \) is transformed into the following one:

\[ K = \frac{\alpha * b * K + a_f \cdot F_{OCн}}{\eta} * \left( \frac{1}{\eta} + \frac{1}{\eta^2} + \ldots + \frac{1}{\eta^T} \right) + \frac{F_O6}{\eta^T}, \]  

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Availability of working capital with the same amount of investments leads to reduction of depreciation, since the latter is calculated only for the fraction containing fixed assets. At the same time when calculating NPV we add the discounted value of working capital to the resources. The economic intuition of this component is in the fact that working capital is used after the end of fixed assets’ life.

Let us give an example, when in addition to the parameters taken from the previous calculations it is assumed that working capital is equal to the fixed assets, that is $\lambda = 1$.

In this case, the equation for determining the rates takes the following form:

$$\eta^{10} = 0.1 \ast \eta^{10-1} + 0.5.$$

Below we present the results of the corresponding iterative process:

1. $\eta_1 = 1.05; \Delta_1 = 1.6289 - 1.7578 = -0.1289.$
2. $\eta_2 = 1.06; \Delta_2 = 1.7908 - 1.8180 = -0.0272.$
3. $\eta_3 = 1.07; \Delta_3 = 1.9671 - 1.8816 = 0.0855.$

Thus, we can conclude that the growth rates take the value equal to $= 1.06$ (6% annual growth).

Availability of working capital leads ceteris paribus to decrease in the growth rates, since flow of depreciation as a direct investment source reduces. For 100% fixed assets growth rates were 8%. Depreciation turnover is an important factor intensifying economic growth dynamics.

We analyzed the cases of dynamics when the acquisition of new assets is made from own funds. At the same time important is the development where the investment resources are involved, that is where the borrowed funds are used.

Let us consider a partial version of investment into funds renewal at the expense of borrowed funds, which must be returned within one year with interest (i).

Analysis of the relevant resource flows shows that in this case of investing the rates are determined from the following equation:

$$(1 + ai)\eta^{T-1} = (\alpha \ast r + a_p) \ast (1 + \eta + ... + \eta^{T-1}). \tag{8}$$

For the specification of this equation, let us consider raising funds at 10% per annum. Then equation (8) takes the following form:

$$1.05\eta^9 = 0.15 \ast \frac{\eta^{10-1}}{\eta^{-1}}.$$

Let us give the final fragments of the iterative process for rates calculating in this equation:

1. $\eta_1 = 1.08; \Delta_1 = 2.1589 - 2.2357 = -0.0768.$
2. $\eta_2 = 1.09; \Delta_2 = 2.3674 - 2.3665 = 0.0009.$

Thus, borrowing funds compared to the development from own resources leads to increase in the absolute annual growth rate from 8% to 9%.

Analysis of equation (8) shows that for the evaluation of fundraising rationality an essential role is played by the ratio between the indicator of assets profitability and the interest rate on borrowing. An important issue is the definition of acceptable lending rate, at which rates are still not slowing down the company’s development.

As an example we will consider an option where $i = 0.16$ (16% per annum). Calculations are made for the equation obtained after some arithmetic operations:

$$\eta^{10} = 0.1389 \ast \frac{\eta^{10-1}}{\eta^{-1}} \ast \eta.$$
The relevant calculations to determine the growth rate we are presented below:

1. \( \eta_1 = 1.05; \Delta_1 = 1.6289 - 1.8342 = -0.2052. \)
2. \( \eta_2 = 1.06; \Delta_2 = 1.7908 - 1.9407 = -0.1499. \)
3. \( \eta_3 = 1.07; \Delta_3 = 1.9671 - 2.0533 = -0.0862. \)
4. \( \eta_4 = 1.08; \Delta_4 = 2.1589 - 2.1684 = -0.0095. \)

Our calculation shows that with the taken interest rate borrowing does not change the growth rates. In other words, borrowing funds for the considered scenario of development makes sense only if the interest rate is maximum 1.6 times higher than return on assets. If the interest rate is higher then borrowing will lead to slowdown in growth rates.

Noteworthy is the version of interest-free loans \((i = 0).\) In this case, equation (8) takes the following equivalent form:

\[
\eta^T = (\alpha r + a_e) (1 + \eta + ... + \eta^{T-1}) \eta. \tag{9}
\]

The calculation shows that in this case the growth rates increase to 15%, that is provide significant intensification of development.

It should be mentioned that to determine the NPV for the case of borrowed funds relation (5) is modified:

\[
K = \left( \frac{R}{\eta} + \frac{R}{\eta^2} + ... + \frac{R}{\eta^T} \right) \frac{\eta}{1 + \alpha i}. \tag{10}
\]

The conclusion is that to determine the NPV a more complex relationship that the discounted cash flows is used. In equation (10) discounted cash flows are multiplied by a certain factor, which, in particular, depends on the growth rates and the interest rates on borrowed funds.

Overall, we can conclude that the considered method of quantitative fixed assets dynamics should be considered rather promising. The method is based on the schemes of fixed assets movement, and on the approach developed to obtain the equations for determining rates based on a number of technical and economic parameters.

The proposed approach allows us to deeper identify patterns of production systems, and use them to solve practical problems, including the choice of sources of financial support, investment decision making, forecasting, and others.

**Conclusions**

Focusing on modeling the growth rate of fixed assets, which are caused by a number of technical and economical parameters, should be considered as an effective approach to the analysis of the production processes dynamics. This enables us to justify in a greater extent the investment decisions, particularly those related to the implementation of investment projects at the level of local production systems.

**Prospects for future research**

In the future it is planned to adjust the obtained quantitative dependences with regard to probabilistic characteristics of primary parameters used in the modeling of production processes dynamics.