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DEVELOPMENT OF VHDL-AMS MODEL OF INTEGRATED MICROACCELEROMETER WITH SIGMA-DELTA CONTROL

In the paper, VHDL-AMS model of microaccelerometer with sigma-delta control for computer-aided design has been developed. The created model has made possible the simulation of dynamic characteristics of the integrated capacitive microaccelerometer, signal processing and digitizing, simulation of force feedback control using sigma-delta technique on the applied force of acceleration and also to perform the analysis of the device at the behavioral level of computer-aided design.

Key words: microelectromechanical systems (MEMS), integrated microaccelerometer, acceleration, analogue-to-digital converter (ADC), sigma-delta force feedback control, VHDL-AMS model, behavioral analysis, hAMSter software.

Розроблено VHDL-AMS модель мікроакселерометра з сигма-дельта контролем для системи автоматизованого проектування. Створена модель зробила можливим моделювання динамічних характеристик інтегрованого ємнісного мікроекселерометра, обробку сигналу і оцифровування, моделювання контролю force feedback, застосовуючи сигма-дельта методику на прикладну силу прискорення, а також для проведення аналізу приладу на рівні поведінки системи автоматизованого проектування.

Ключові слова: мікроелектромеханічна система (МЕМС), інтегрований мікроаксе-лерометр, прискорення, аналого-цифровий перетворювач (АЦП), сигма-дельта force feedback (форс фідбек) контроль, VHDL-AMS модель, аналіз поведінки, програмне забезпечення hAMSter.

Introduction

MEMS inertial sensors include accelerometers and gyroscopes which are widely used in many areas: in cars for airbag deployment, consumer electronics applications such as smart phones and in industrial, aerospace and defense applications such as oil exploration, structural health monitoring for bridges, and inertial measurement units for navigation [1-3].

In Fig. 1, a typical design for a surface micromechanical capacitive sensing element structure of the integrated microaccelerometer is shown. The sensing element consists of a proof mass suspended above a substrate by springs. The proof mass is equipped by a number of sense and force comb finger units. Each comb finger unit contains a movable sense or force finger (connected to the proof mass) that is placed between two fixed fingers.

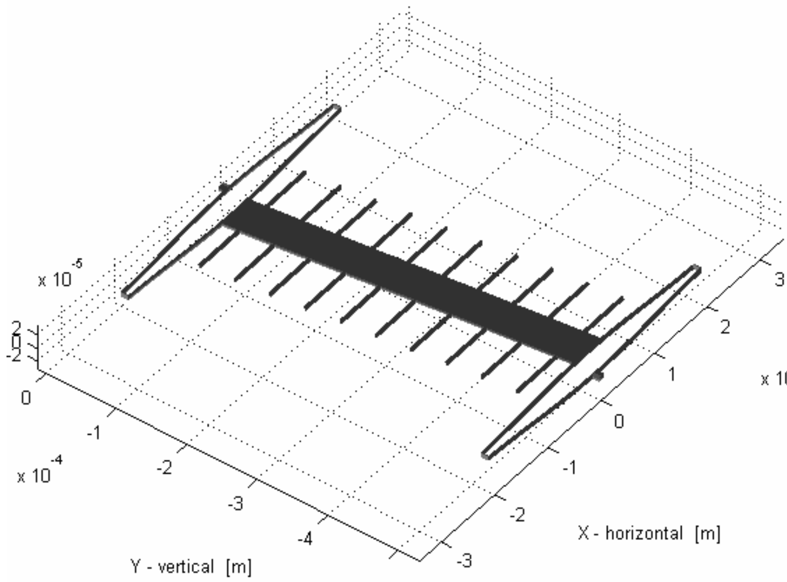

Fig. 1. Model of MEMS capacitive comb-drive accelerometer

1. Sigma-delta modulation technique

The topology of a closed-loop digital MEMS accelerometer is inspired by sigma-delta modulators. Analogue-to-digital converters (ADCs) can be divided into two categories: Nyquist-rate ADCs, oversampling ADCs, such as sigma-delta modulator, can achieve higher resolution and release critical requirements on the IC fabrication process by sacrificing the signal bandwidth. Oversampling and noise shaping are the two main techniques employed in the sigma-delta modulators to achieve their advantages. The oversampling technique makes the noise spread over a wider frequency range; while the noise shape dynamically decreases the noise in the signal band; therefore, higher resolution is available.

High-performance MEMS sensors usually take advantage of a sigma-delta force feedback control strategy to improve linearity, dynamic range, and bandwidth, and provide direct digital output in the form of pulse density modulated bitstream, which can interface with a digital signal processor. This approach has been applied to MEMS accelerometers and gyroscopes.

The diagram of a second-order electromechanical sigma-delta accelerometer is shown in Fig. 2.

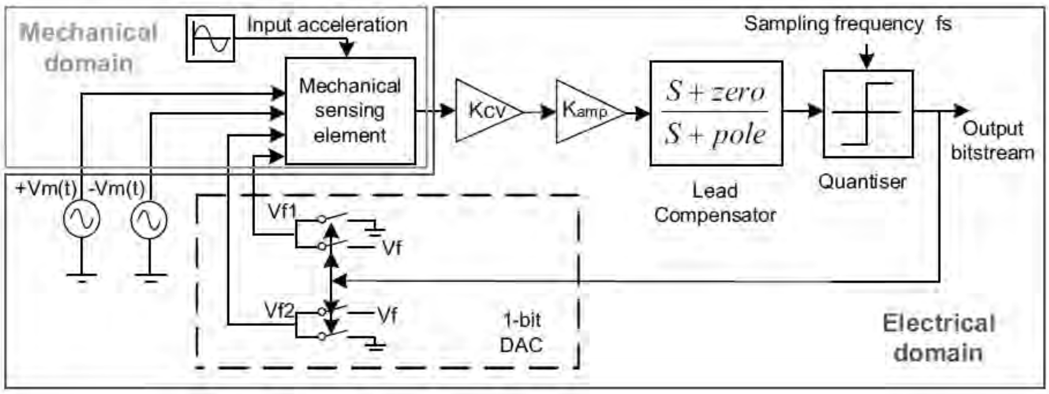

Fig. 2. Second-order electromechanical sigma-delta accelerometer

The mechanical sensing element is followed by the signal pick-off circuit which is represented by a gain block K_{cv}. K_{amp} is the gain of the voltage booster amplifier following the pick-off stage. $V_{f1}(t)$ and $V_{f2}(t)$ are the feedback voltages obtained from the DAC to generate electrostatic feedback force in the mechanical sensing element, and $V_m(t)$ is a high frequency modulation voltage. A lead compensator is required to stabilize the system. A one-bit quantiser is used to oversample and convert the analogue voltage to a pulse density modulated digital signal. f_s is the oversampling frequency. If the signal bandwidth of the system is f_0, the oversampling ratio (OSR) of the system is given by:

$$OSR = \frac{f_s}{2f_0} \quad (1)$$

The mechanical sensing element of the MEMS sigma-delta modulator is used as a loop filter. This is because the sensing element is conventionally approximated by a second-order mass-damper-spring system which performs a similar function to that two cascaded integrators in a typical second-order electronic sigma-delta modulator. Thus, in such a configuration, dynamics of the mechanical sensing element limit the performance of the system. The mechanical sensing element in the closed-loop is usually modeled by the following equation:

$$Ma_{in} + F_{feedback} = M\frac{d^2x}{dt^2} + D\frac{dx}{dt} + Kx \quad (2)$$

Where x is the relative displacement of the proof mass with respect to the substrate, a_{in} is the input acceleration, and $F_{feedback}$ is the feedback force. M, K, D are lumped parameters which represent proof mass (kg), spring constant (N/M) and damping coefficient (Ns/m) respectively.

The proof mass is suspended by four cantilever beam springs and equipped with movable comb fingers which are placed between the fixed fingers as the common centre electrode to form capacitance bridges. Such a constructed mechanical sensing element can detect a differential change in capacitance caused by the displacement of movable fingers, and convert it to voltage by associated interface circuits. Among these capacitance bridges, most capacitance groups act as sensing capacitance (sense units) and a few other capacitance groups (force units) are used to generate electrostatic feedback force. In the closed-loop operation, feedback voltages ($V_{f1}(t)$ and $V_{f2}(t)$) are applied to the fixed fingers in each force unit such that the resulting electrostatic force pulls the moving proof mass back to its original position. Assuming the displacement of the force finger is much smaller than the initial gap (G_2) between the force finger and the fixed fingers in a force unit, the expression of the feedback electrostatic force is given by:

$$F_{feedback} = \frac{N_f \varepsilon_0 L_{ff} T}{2G_2^2}\left(V_{f1}^2 - V_{f2}^2\right) \quad (3)$$

where ε_0 is dielectric constant, N_f is the number of the force fingers, L_{ff} and T are the length and the thickness of the force fingers respectively.

Now, we calculate the lumped parameters, mass (M), damping coefficient (D) and spring constant (K) in eq., according to layout of the mechanical sensing element.

The proof mass (M) can be calculated by assuming it is a single polysilicon with density $\rho = 2330$ kg/m^3.

$$M = \rho\left(V_{mass} + V_{fingers}\right) = \rho\left(W_{pm}L_{pm} + \left(N_s + N_f\right)L_{sf}W_{sf}\right)T =$$
$$= 2330\times\left(120\times10^{-6}\times450\times10^{-6} + (54+4)\times150\times10^{-6}\times2\times10^{-6}\right)\times2\times10^{-6} = 3.32\times10^{-10}\,kg, \quad (4)$$

where V_{mass} and $V_{fingers}$ are the volumes of the proof mass and movable fingers respectively.

The suspension system of the mechanical sensing element consists of four cantilever springs which are shown in Figure. The expression for the spring constant of each cantilever is given by:

$$K_{cantilever} = \frac{12EI_s}{L_s^3} = \frac{EW_s^3T}{L_s^3} = \frac{190\times10^9\times\left(2\times10^{-6}\right)^3\times2\times10^{-6}}{\left(176\times10^{-6}\right)^3} = 0.56N/M\,, \quad (5)$$

where E=170×10^9 N/m^2 is the Young's modulus for polysilicon. I_s is the moment of inertia of the cantilever which is equal to $\frac{W_s^3T}{L_s^3}$. W_s, L_s, and T represent the width, length, and thickness of the cantilever spring respectively. Because the proof mass is supported by four cantilevers of equal dimensions, each spring shares ¼ of the total force load. Thus, the total mechanical spring constant is $4K_{cantilever}$.

$$K_{mechanical} = 4\times K_{cantilever} = 4\times0.56 = 2.24N/M \quad (6)$$

The calculated spring constant above does not take into account the electrostatic spring softening effect. In a sense unit shown in Figure, when a high frequency square modulation voltage $V_m(t)$ is applied to the fixed fingers, electrostatic forces are generated on the sense finger that lead to a change of the actual spring constant from its mechanical value. This phenomenon is regarded as electrostatic spring softening and is also included in our mechanical sensing element model. The net force on the sense finger (F_s) is given:

$$F_e = F_{e1} - F_{e2} = \frac{\varepsilon_0 A V_m^2}{2}\left[\frac{1}{(G-x)^2} - \frac{1}{(G+x)^2}\right] \quad (7)$$

where F_{e1} and F_{e2} are electrostatic forces, G is the initial gap between the sense finger and fixed fingers, x is the displacement of the sense finger, A is the area of the sense finger sidewall ($A = L_{sf}T$), ε_0 is dielectric constant and V_m is the amplitude of the modulation voltage (1V in this design).

Assuming $x << G$ and considering there are N_s sense units, the electrostatic spring constant can be given by:

$$K_e = N_s\left(\frac{d(F_e)}{dx}\right) = -N_s\left(\frac{2\varepsilon_0 L_{sf} T V_m^2}{G^3}\right) =$$
$$= -54\times\frac{2\times 8.85\times 10^{-12}\times 150\times 10^{-6}\times 2\times 10^{-6}\times 1^2}{\left(1.3\times 10^{-6}\right)^3} = -0.13 N/M \quad (8)$$

Consequently, the effective spring constant is equal to:

$$K = K_{mechanical} + K_e = 2.24 - 0.13 = 2.11 N/M \quad (9)$$

For a displacement of the movable structures, the gas in the gaps between the movable and the fixed structures is compressed or expanded and begins to stream. For this capacitive mechanical sensing element, squeeze-film damping between the comb-fingers usually dominates all other forms of damping. Squeeze-film damping can be modeled by assuming the Hagen-Poiseulle flow between comb-fingers. Neglecting the fringing fields, the damping coefficient is given by:

$$D = 14.4\left(N_f + N_s\right)\mu L_{sf}\left(\frac{T}{G}\right)^3 =$$
$$= 14.4\times(54+4)\times 1.85\times 10^{-5}\times 150\times 10^{-6}\left(\frac{2\times 10^{-6}}{1.3\times 10^{-6}}\right)^3 = 8.44\times 10^{-6} N\cdot s/m, \quad (10)$$

where L_{sf} and T represent the length and thickness of the movable fingers, G is the initial gap between fixed fingers and movable fingers, and μ is the viscosity coefficient of the air.

From the calculated lumped parameters, we can obtain some performance parameters. Resonant frequency f_0: Recalling equation, the resonant frequency (f_0) can be calculated from the mass (M) and the effective spring constant (K):

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\sqrt{\frac{2.11}{3.32\times 10^{-10}}} = 12.6 KHz\,. \quad (11)$$

There is only one resonant mode (12.6 kHz) in the lumped mass-damper-spring system. In reality, the mechanical sensing element is a distributed element with many resonant modes. Our proposed distributed sensing element model captures the higher resonant modes of the sensing element and provides more accurate simulation results than the conventional lumped model.

Recalling equation, the quality factor (Q) of the mechanical sensing element is derived from the lumped parameters, proof mass (M), spring constant (K) and damping coefficient (D):

$$Q = \frac{\sqrt{KM}}{D} = \frac{M\omega_0}{D} = \sqrt{\frac{2.11\times 3.32\times 10^{-10}}{8.44\times 10^{-6}}} = 3.13\,. \quad (12)$$

The dynamic response of the mechanical sensing element can be divided into three types according to the value of quality factor (Q): if $Q < 0.5$, the sensing element is over-damped; if $Q = 0.5$, it is critically damped; otherwise, it is under-damped.

2. Development of VHDL-AMS model of the integrated microaccelerometer with sigma-delta control

At the schematic level of MEMS design, behavioural models can be developed. The peculiarity of such models is that they can contain information from the different scientific and engineering areas. As example, in the model of the integrated microaccelerometer manufactured by using micromachining technologies, magnitudes from mechanics, electrical engineering and electronics are used. Extension of VHDL standard to VHDL-AMS (VHSIC Hardware Description Language Analog-Mixed Signals) makes possible to create both digital- and analog- and mixed-signal behavioural models, which use not only electrical signals, but also optical, chemical, thermal, mechanical, etc [4-7]. In Fig. 3, the behavioral model of the the integrated microaccelerometer with sigma-delta control developed in VHDL-AMS is presented.

```
-- VHDL-AMS model of integrated microaccelerometer with sigma-delta control --
library ieee, disciplines;
use disciplines.kinematic_system.all;
use disciplines.electromagnetic_system.all;
use ieee.math_real.all;
entity sensing_element is
 generic( --Dimension of mechanical sensing element--
                Wpm: real := 120.0e-6; Lpm: real := 450.0e-6; T: real := 2.0e-6;
                Ls: real := 176.0e-6; Ws: real := 2.0e-6; Lsf: real := 150.0e-6;
                Wsf: real := 2.0e-6; Lff: real := 150.0e-6; Wff: real := 2.0e-6;
                G: real := 1.3e-6; G2: real := 1.3e-6; Ns: real := 54.0;
                Nf: real := 4.0; Vm: real := 1.0);
port ( quantity ain: in acceleration; -- Input acceleration
          quantity Vf1: in real; -- Feedback voltage to top fixed fingers in force units
          quantity Vf2: in real; -- Feedback voltage to bottom fixed
          quantity pos: out displacement); --Displacement of proof mass
end entity sensing_element;
architecture behav of sensing_element is
        quantity M: real;
        quantity K: real;
        quantity Kmechanical: real;
        quantity Ke: real;
        quantity D: real;
        quantity Ff: real;
        constant PHYS_RHO_POLY: real := 2330.0;   -- 2330 kg/m^3
        constant PHYS_E_POLY: real := 170.0e9;   -- 170*10^9 N/m^2
   constant PHYS_EPS0: real := 8.85e-12;   -- 8.85*10^-12 N/m^2
begin
        --Mass of sensing element--
        M == PHYS_RHO_POLY*(Wpm*Lpm*T+(Ns+Nf)*Lsf*Wsf*T);
        --Mechanical spring--
        Kmechanical == 4.0*PHYS_E_POLY*Ws*Ws*Ws*T/(Ls*Ls*Ls);
        --Electrostatic spring--
        Ke == -1.0*Ns*(2.0*PHYS_EPS0*Lsf*T*Vm*Vm)/(G*G*G);
        --Effective spring constant--
        K == Kmechanical*Ke;
        --Damping coefficient--
        D == 14.4*(Ns+Nf)*1.85e-5*T*Lsf*Lsf*Lsf/(G*G*G);
        --Feedback force--
        Ff == 0.5*Nf*PHYS_EPS0*Lff*T*(Vf1*Vf1-Vf2*Vf2)/(G2*G2);
        --Behaviour of mechanical sensing element--
        M*pos'DOT'DOT+D*pos'DOT+K*pos == M*ain+Ff;
end architecture behav;
```

```
entity compensator is
generic (
   K: real := 4.0; -- gain
   Fp: real := 2.0e+4; -- pole frequency
   Fz: real := 1000.0 -- zero frequency
);
port (terminal ip, op : electrical);
end entity compensator;

architecture behavioral of compensator is
          quantity vin across ip to electrical_ground;
          quantity vout across op to electrical_ground;
          constant num : real_vector (1 to 2) := (Fz, 1.0);
          constant den : real_vector (1 to 2) := (Fp, 1.0);
begin
    vout == K * vin'ltf(num, den);
end architecture behavioral;

library ieee, disciplines;
use disciplines.electromagnetic_system.all;
use ieee.math_real.all;
use ieee.std_logic_1164.all;

entity quantizer is
   generic(Fs: real := 0.0); --threshod : real := 0.0);
   port (
       signal clk : in std_logic;
       terminal ip : electrical; --input analog signal
       signal op : out std_logic); --output digital signal
end entity quantizer;

architecture bhv of quantizer is
          quantity vip across ip;
          begin
                    process(clk)
                              begin
                                        if(rising_edge(clk)) then
                                                  if vip > Fs then --threshod then
                                                            op <= '1';
                                                  else
                                                            op <= '0';
                                                  end if;
                                        end if;
                    end process;
          end architecture bhv;
entity DAC is
          port (signal ip : in std_logic; -- input digital signal
                     --quantity fin: force;
                     terminal op1, op2: kinematic); -- output analog signal
end entity DAC;
architecture bhv of DAC is
          signal Fe : force := 0.0;
          quantity F through op1;
begin
```

```
            Fe <= 35.0E-6 when ip <= '0' else -35.0E-6;
            F == -Fe;--+fin;
end architecture bhv;
entity gain is
            generic (K: real := 10.0e5);
            port (terminal op_inp: electrical;
                          terminal op_inn: electrical;
                          terminal op_outp: electrical);
end entity gain;
entity a_source is -- acceleration
  generic (MAG : real := 1.0*9.8; FREQ: real := 1000.0);
  port (quantity op: out real);
end entity a_source;
architecture sine of a_source is
  begin
            op == MAG * sin(MATH_2_PI*FREQ*NOW);
end architecture sine;

entity test_ACCELEROMETER is
end entity test_ACCELEROMETER;
architecture testbench of test_ACCELEROMETER is
            quantity a: acceleration;
            quantity d: displacement;
            quantity V1, V2, vp, vb, como: real;
            signal output: std_logic;
   constant PHYS_GRAVITY: real := 9.8; -- 9.8 m/s^2
begin
  Acceleration: entity a_source(sine)
                                        generic map (MAG=>1.0*PHYS_GRAVITY, FREQ=>1000.0)
                                        port map (op=>a);
  Sensing: entity sensing_element
                          generic map (Wpm=>120.0e-6, Lpm=>450.0e-6, T=>2.0e-6,
                                                    Ls=>176.0e-6, Ws=>2.0e-6, Lsf=>150.0e-6,
                                                    Wsf=>2.0e-6, Lff=>150.0e-6, Wff=>2.0e-6,
                                                    G=>1.3e-6, G2=>1.3e-6, Ns=>54.0,
                                                    Nf=>4.0, Vm=>1.0)
                          port map (ain=>a, Vf1=>V1, Vf2=>V2, pos=>d);
-- Pick_off_gain: entity gain
--                                      generic map (K=>41.0e6)
--                                      port map (ip=>d, op=>vp);
-- Boost_gain: entity gain
--                                generic map (K=>37)
 --         port map (ip=>vp, op=>vb);
 Compensation: entity compensator
                                 generic map (Fz=>5000.0, Fp=>250000.0)
            port map (ip=>vb, op=>como);
 Q:                             entity quantizer
                                generic map (Fs=>2048.0*256.0*2.0)
                                port map (ip=>como, op=>output);
 DAC:  entity DAC(bhv)
             port map (ip=>output, op1=>V1, op2=>V2);
end architecture testbench;
```

Fig. 3. VHDL-AMS model of the integrated microaccelerometer with sigma-delta control

3. Computer simulation results

The developed VHDL-AMS model makes possible to simulate dynamic characteristics of the integrated capacitive microaccelerometer, signal processing and digitizing, simulation of force feedback control using sigma-delta technique on the applied force of acceleration. The simulation results at the applied sinusoidal acceleration 1 g are graphically illustrated in Fig. 4.

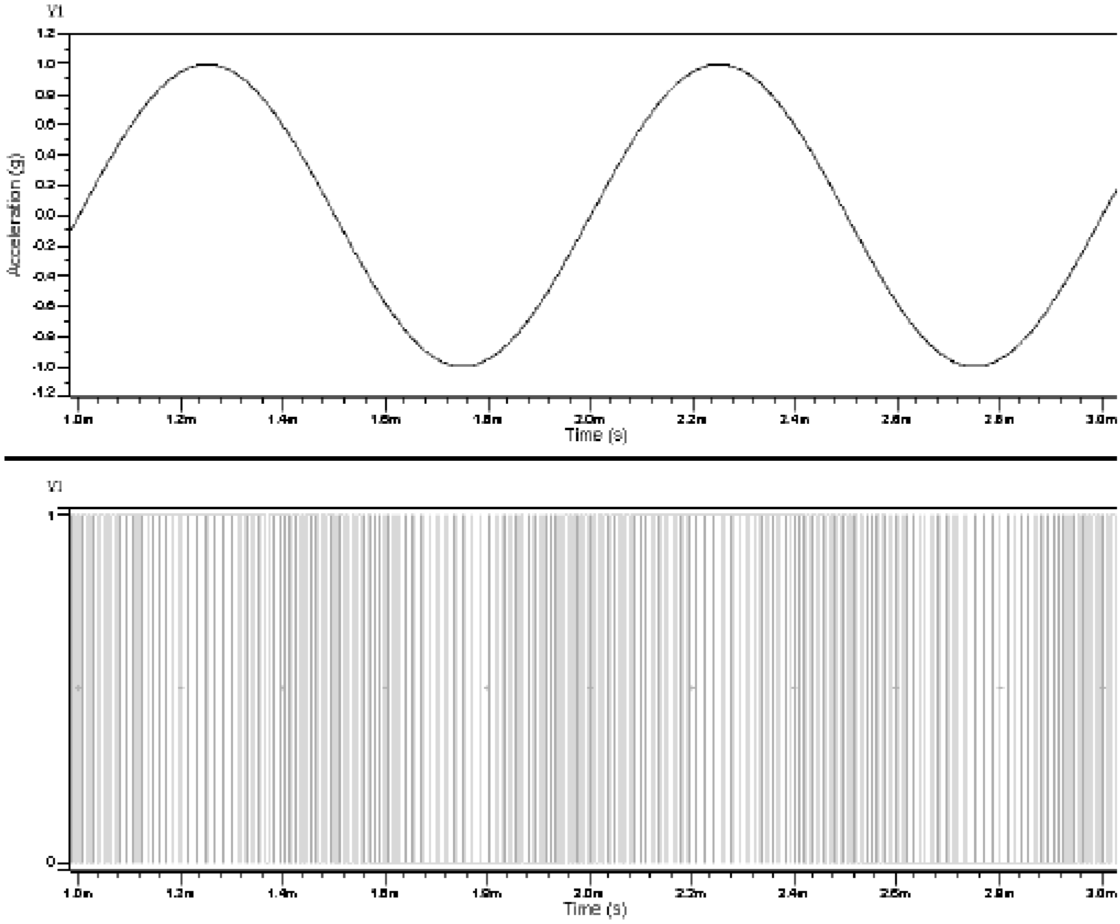

Fig. 4. Simulation results

Conclusion

VHDL-AMS model of microaccelerometer with sigma-delta control for computer-aided design has been developed. The created model has made possible the simulation of dynamic characteristics of the integrated capacitive microaccelerometer, signal processing and digitizing, simulation of force feedback control using sigma-delta technique on the applied force of acceleration and also to perform the analysis of the device at the behavioral level of computer-aided design.

1. Kanchan Sharma, Isaac G. Macwan, Linfeng Zhang, Lawrence Hmurcik, Xingguo Xiong. Design Optimization of MEMS Comb Accele rometer, D epartment of Ele ctrical and Com puter Eng ineering. – pp. 10.2. To lga Ka ya, B ehrouz Shiari, Kev in P etsch1 and Da vid Yates. Des ign of a MEMS Capa citive Comb-drive Accelerometer. Central Michigan University, University of Michigan, 2012. – pp. 6. 3 . Akila Kannan. Design and Model ing of a MEMS-Bas ed Acce lerometer with Pull In A nalysis. Thesis, University of British Columbia, 1 49, pp. 149. 4. hAMSter Softw are fo r VHDL- AMS Simulatio ns, http://www.theoinf.tu-ilmenau.de/~twangl/VHDL-AMS_online_en/Home.html. 5. Peter J. Ashenden ED A CONSULTANT, ASHENDEN D ESIGNS PT Y. LTD., "V HDL Tutorial", Elsevier Science 2004. – pp. 84. 6. Standar d VH DL Analog and Mixed-Signal Extensions - Packages for Multiple Energy D omain S upport – 2003. – p p. 21. 7. VHDL 1076.1: Analog Extensions to VHDL, Ernst Christen, Analogy Inc. – April 1997. – pp. 9 .