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SIMPLE METHOD OF OBTAINING OF LOGIC FUNCTIONS POLYNOMIALS WITH GIVEN POLARITY OF VARIABLES

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This article presents a simple numeric set-theoretical method of obtaining of the logic functions Fixed Polarity Reed-Muller polynomials (including of Zhegalkin polynomial) with arbitrarily given polarity of *n* variables. The advantages of the suggested method are illustrated by the examples.

Key words: logic function, ternary conjuncterm, SOP, numeric set-theoretical method, STF, numeric Zhegalkin polynomial, numeric Fixed Polarity Reed-Muller (FPRM) polynomial.

Описано простий числовий теоретико-множинний метод одержання поліномів Ріда-Маллера з фіксованою полярністю (зокрема поліном Жеґалкіна) логікових функцій від *n* змінних. Переваги методу проілюстровано на прикладах.

Ключові слова: логікова функція, трійковий кон'юнктерм, ДНФ, теоретико-множинний метод, ТМФ, числовий поліном Жеґалкіна, числові поліноми Ріда-Маллера з фіксованою полярністю.

Introduction

Significant stage of designing of any digital device (DD) is logic synthesis that involves building of a structural model of the device on the basis of optimization method – that of decomposition and/or minimization of logic function or system of functions which describe its work. Generally structural optimization is done on the basis of two-level synthesis with the use of logic units (LUs) of the AND-OR-type, when synthesized digital device is described by the function in Sum-Of-Product form (SOP). It provides that the input digital device serves variables in direct and in inverse proportions, forming a conjunctive terms (conjuncterms) of a given function.

Lately more attention is paid to LUs of the AND-EXOR-type when DD is described by the function in polynomial form (Exclusive Sum-Of-Product form, ESOP) in which instead of disjunction we use the mod-2-sum and constant **1**. In [1-4] it is shown that the DD built on LUs and ESOP, if compared with traditional LUs of the AND-OR, have certain advantages. It is easier to test and diagnose them and for realization of some classes of functions such LUs are required (on average) comparatively less.

In connection with the search for the optimal solution of the logic synthesis problem of DD (minimum number conjuncterms and number of literals) arises a need to convert (canonical) SOP into ESOP, in particular, into Zhegalkin polynominal or into Fixed Polarity Reed-Muller polynominal (FPRM) [1, 2]. In the first case all literals of function in conjuncterms have no sign of inversion (so-called positive polarity of variables) and such polynomial for the function of *n* variables is unique, in the second case – some literals have an inversion sign (so-called negative polarity) and the other do not have. Such correspondingly polynomials with different polarity for a function of *n* variables will be 2^n.

Transfomation into ESOP with optimal polarity of variables of the given function leads to solving complex search synthesis problem of combinatorial type. With this purpose a table method on the basis of Karnaugh maps [2,5,6] is used, vector-matrix method [3,4] and on the basis of the determination of so-called FPRM coefficients [7, 8]. They are rather complicated and bulky as to their realization on computer and require interim transformation. The first mentioned method in particular has obvious limits as to the number of variables and instead of the other ones provides getting Kronecker's multiplication with matrices of 2^n order.

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Suggested in this article new method of obtaining of polynomials of the given polarity is based on numerical set-theoretical approach [9,10] and if compared with known methods is simpler for practical realization, in particular, it is easy to realize on computer without any prior transformations.

Main part

It is known that any logic function $f(x_1, x_2, ..., x_n)$ can be described in polinomic form as a mod-2-sum of its conjuncterms. For example, let canonical SOP of the function

$$f = \bar{x}_1\bar{x}_2\bar{x}_3 \vee \bar{x}_1 x_2 x_3 \vee x_1 x_2 \bar{x}_3.$$

In order to get Zhegalkin polynomial of this function it is necessary at first to write down its canonical ESOP, having changed the signs of disjunction (∨) by the signs of the mod-2-sum (⊕) and then to do Reed-Muller expansion using for every *i*-th of inverse variable the expression $\bar{x}_i = x_i \oplus 1$:

$$f = \bar{x}_1\bar{x}_2\bar{x}_3 \vee \bar{x}_1 x_2 x_3 \vee x_1 x_2 \bar{x}_3 = \bar{x}_1\bar{x}_2\bar{x}_3 \oplus \bar{x}_1 x_2 x_3 \oplus x_1 x_2 \bar{x}_3 =$$
$$= (1 \oplus x_1)(1 \oplus x_2)(1 \oplus x_3) \oplus (1 \oplus x_1) x_2 x_3 \oplus x_1 x_2 (1 \oplus x_3) =$$
$$= 1 \oplus x_3 \oplus x_2 \oplus x_1 \oplus x_1 x_2 \oplus x_1 x_3 \oplus x_2 x_3 \oplus x_1 x_2 x_3 \oplus x_2 x_3 \oplus x_1 x_2 x_3 \oplus x_1 x_2 \oplus x_1 x_2 x_3.$$

After removing pairs of equal conjuncterms (as an example, $x_1 x_2 \oplus x_1 x_2 = 0$) we get the searched Zhegalkin polynomial:

$$f = 1 \oplus x_3 \oplus x_2 \oplus x_1 \oplus x_1 x_3 \oplus x_1 x_2 x_3.$$

In the similar way one can get FPRM polinomial of a function with certain polarity of its variables if for some variables we apply the expression $\bar{x} = x \oplus 1$ and for others – $x = \bar{x} \oplus 1$.

If the function *f* which is to be converted into Zhegalkin polynomial or FPRM polynomial is given in SOP it is necessary at first to ortogonalize [3, 4, 9].

The essence of the suggested numerical set-theoretical method of obtaining of given polarity polynomials is in intermediate conversion of each numeric (binary or ternary) conjuncterm of the rank $r \in \{1,2,...,n\}$ of the logic function variales $f(x_1, x_2, ..., x_n)$, which is given in set-theoretical (not necessarily canonical) form (STF Y^1) [9, 10], in some set of binary and/or ternary conjuncterms of certain ranks $r \in \{0,1,2,...,n\}$. The procedure of conversion (algorithm which is discussed further) is done by simple change of certain positions of given (forming) conjuncterms of the STF Y^1 by values from the set $\{0,1,-\}$ that is determined by the given *C*-polarity. The operator of such procedure will be marked by the symbol $\overset{C}{\Rightarrow}$, where $C = \rho_1\rho_2 \cdots \rho_n$, $\rho_i \in \{0,1\}$ – *polarity code* which determines the value of positions of formed ternary conjuncterms of function *f*. The resulting set of conjuncterms (it will be marked as $Y^{\oplus}$) is considered in polynomial format, and pairs of equal elements are removed from it. As a result of this so-called *polynomial set-theoretical form* (PSTF $Y^{\oplus}$) of function *f* is obtained which we will call *numeric Zhegalkin polynomial* if it is formed by the polarity code $C = 11 \cdots 1$, or *numeric FPRM polynomial* if it is formed by the code $C \neq 11 \cdots 1$. In this sense, the conversion into a numerical Zhegalkin polynomial can be considered as a particular case of conversion into a numeric FPRM polynomial with arbitrary polarity code *C*. If necessary, obtaining of an analytical expression of the PSTF $Y^{\oplus}$ can be done by a simple procedure [10]: $(0)_i \to \bar{x}_i$, $(1)_i \to x_i$, $(-)_i \to$ x_i is absent.

The proposed method of obtaining of PSTF $Y^{\oplus}$ for a certain *C*-polarity we will first consider for the function, which is given in canonical STF $Y^1 = \{m_1, m_2, ..., m_s\}^1$, where $m_i = (\sigma_1\sigma_2 \cdots \sigma_n)_i$, $\sigma_j \in \{0,1\}$, – *i*-th binary minterm of function *f*. Algorithm procedure of minterm m_i converting into some set PSTF $Y^{\oplus} = \{\theta_1, \theta_2, ..., \theta_p\}^{\oplus}$, where $\theta_i = (\sigma_1\sigma_2 \cdots \sigma_n)$, $\sigma_j \in \{0,1,-\}$, – *i*-th ternary conjuncterm of function *f*, is executed by the sequence of the following steps:

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step 1: rewrite the code of given polarity $C = \rho_1\rho_2\cdots\rho_n$, $\rho_i \in \{0,1\}$, taking it as the first element $(\sigma_1\sigma_2\cdots\sigma_n)$, $\sigma_j \equiv \rho_j$, of the desired PSTF $Y^{\oplus}$; in it we distinguish (eg, bold type) the positions that differ it from the polarity of the code *C*;

step 2: rewrite the obtained first element is sequence for each change for the dash (–) one of its chosen positions, beginning, for example, with the least significant position;

step 3: perform a similar procedure with the first element sequentially replacing two selected positions for the dashes (–);

etc. at each following step a similar procedure is performed to replace the selected positions of the first element for one dash (–) more up to full substitution of dashes (–).

Algorithm for converting of binary minterms of canonical STF Y^1 into PSTF $Y^{\oplus}$ for a given *C*-polarity will be discussed in detail.

Let *k* be the number of significant positions of binary minterm that differ it from the given of polarity code *C*. Then converting procedure of minterm will have $k+1$ steps and the formed PSTF $Y^{\oplus}$ thus consists of the subsets of conjuncterms with its own rank *r*, namely: on the step 1 we will have $r = n$, on the step 1 we will have $r = n-1$, ..., on the step $k+1$ we will have $r = n-k$. As a result of this the power (number of conjuncterms) of each formed subset depends on *k* and number *q* of made of dashes (–) in conjuncterms and its value is determined by combinatorial $C_k^q = \dfrac{k!}{q!(k-q)!}$, $q = 0,1,...,k$. Accordingly, the power of PSTF $Y^{\oplus}$ is equal to 2^k, that is $C_k^0 + C_k^1 + ... + C_k^k = 2^k$. We illustrate this by the example of obtaining of numeric Zhegalkin polynomials and numeric FPRM polynomials, for example, with (101)-polarity for binary minterms (000), (010) and (101) of function $f(x_1, x_2, x_3)$. Applying the relevant operators $\overset{111}{\Rightarrow}$ and $\overset{101}{\Rightarrow}$ we have:

$$(000)\overset{111}{\Rightarrow}\{\underbrace{(\mathbf{111})}_{C_3^0},\underbrace{(11-),(1-1),(-11)}_{C_3^1},\underbrace{(1--),(-1-),(--1)}_{C_3^2},\underbrace{(---)}_{C_3^3}\}^{\oplus},$$

$$(000)\overset{101}{\Rightarrow}\{\underbrace{(\mathbf{101})}_{C_2^0},\underbrace{(10-),(-01)}_{C_2^1},\underbrace{(-0-)}_{C_2^2}\}^{\oplus};$$

$$(010)\overset{111}{\Rightarrow}\{\underbrace{(\mathbf{1}1\mathbf{1})}_{C_2^0},\underbrace{(11-),(-11)}_{C_2^1},\underbrace{(-1-)}_{C_2^2}\}^{\oplus},$$

$$(010)\overset{101}{\Rightarrow}\{\underbrace{(\mathbf{101})}_{C_3^0},\underbrace{(10-),(1-1),(-01)}_{C_3^1},\underbrace{(1--),(-0-),(--1)}_{C_3^2},\underbrace{(---)}_{C_3^3}\}^{\oplus};$$

$$(101)\overset{111}{\Rightarrow}\{\underbrace{(1\mathbf{1}1)}_{C_1^0},\underbrace{(1-1)}_{C_1^1}\}^{\oplus},\quad (101)\overset{101}{\Rightarrow}\{\underbrace{(101)}_{C_0^0}\}^{\oplus}.$$

Below we give analytical expressions of minterms $\bar{x}_1\bar{x}_2\bar{x}_3$, $\bar{x}_1 x_2\bar{x}_3$, $x_1\bar{x}_2 x_3$ and the corresponding polynomials of (111)- and (101)-polarity:

$$\bar{x}_1\bar{x}_2\bar{x}_3 = x_1x_2x_3 \oplus x_1x_2 \oplus x_1x_3 \oplus x_2x_3 \oplus x_1 \oplus x_2 \oplus x_3 \oplus 1,\quad \bar{x}_1\bar{x}_2\bar{x}_3 = x_1\bar{x}_2x_3 \oplus x_1\bar{x}_2 \oplus \bar{x}_2x_3 \oplus \bar{x}_2;$$

$$\bar{x}_1x_2\bar{x}_3 = x_1x_2x_3 \oplus x_1x_2 \oplus x_2x_3 \oplus x_2,\quad \bar{x}_1x_2\bar{x}_3 = x_1\bar{x}_2x_3 \oplus x_1\bar{x}_2 \oplus x_1x_3 \oplus \bar{x}_2x_3 \oplus x_1 \oplus \bar{x}_2 \oplus x_3 \oplus 1;$$

$$x_1\bar{x}_2x_3 = x_1x_2x_3 \oplus x_1x_3,\quad x_1\bar{x}_2x_3 = x_1\bar{x}_2x_3.$$

On this occasion we note that analytical expressions of conjuncterms can be converted into their set-theoretical equivalent by the rule [10]: $\bar{x}_i \to (0)_i$, $x_i \to (1)_i$, x_i is absent $\to (-)_i$.

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As a result of converting of binary minterms of function *f*, that is given by canonical STF Y^1, the set PSTF $Y^{\oplus}$ is formed, which can be simplified by the removal from it pairs of identical elements.

The conversion "canonical STF $Y^1 \overset{C}{\Rightarrow}$ PSTF $Y^{\oplus}$" will be illustrated on the example of earlier considered function $f=\bar{x}_1\bar{x}_2\bar{x}_3 \vee \bar{x}_1 x_2 x_3 \vee x_1 x_2 \bar{x}_3$ for the case of a numeric Zhegalkin polynomial. For this we write down its canonical STF Y^1 and doing the procedure of converting minterms we get the set $Y^{\oplus}$. This one can be simplified by eliminating (see further deletion) pairs of identical conjuncterms. To verify the results, the obtained PSTF $Y^{\oplus}$ will be converted into an analytical expression:

$$Y^1=\{(000),(011),(110)\}^1 \overset{111}{\Rightarrow}\{\begin{pmatrix}\cancel{111}\\ \cancel{11-}\\ 1-1\\ \cancel{-11}\\ 1--\\ -1-\\ --1\\ ---\end{pmatrix},\begin{pmatrix}\cancel{111}\\ \cancel{-11}\end{pmatrix},\begin{pmatrix}111\\ \cancel{11-}\end{pmatrix}\}^{\oplus} \Rightarrow \{\begin{pmatrix}1-1\\ 1--\\ -1-\\ --1\\ ---\end{pmatrix},(111)\}^{\oplus}=$$

$$=\{(1-1),(1--),(-1-),(--1),(---),(111)\}^{\oplus} \Rightarrow x_1x_3 \oplus x_1 \oplus x_2 \oplus x_3 \oplus 1 \oplus x_1x_2x_3.$$

The described method of obtaining of FPRM polynomials of a given *C*-polarity for minterms is suitable to convert conjuncterms of arbitrary ranks $r \in \{1,2,\ldots,n\}$ of function *f*. In this case, dashes (–), symbolizing absorbed positions of ternary conjuncterms STF Y^1, are moved to the same positions of conjuncterms PSTF $Y^{\oplus}$. The truth of this statement will be shown on an example of conjuncterm of 1-rank $(--0)$, having converted its minterms, for example, into numeric Zhegalkin polynomials:

$$(--0)=\{(000),(010),(100),(110)\}^1 \overset{111}{\Rightarrow}\{\begin{pmatrix}\cancel{111}\\ \cancel{11-}\\ \cancel{1-1}\\ \cancel{-11}\\ \cancel{1--}\\ \cancel{-1-}\\ --1\\ ---\end{pmatrix},\begin{pmatrix}\cancel{111}\\ \cancel{11-}\\ \cancel{-11}\\ \cancel{-1-}\end{pmatrix},\begin{pmatrix}\cancel{111}\\ \cancel{11-}\\ \cancel{1-1}\\ \cancel{1--}\end{pmatrix},\begin{pmatrix}\cancel{111}\\ \cancel{11-}\end{pmatrix}\}^{\oplus}=\{\begin{pmatrix}--1\\ ---\end{pmatrix}\}^{\oplus}.$$

So, we have $(--0)\overset{111}{\Rightarrow}\{(--1),(---)\}^{\oplus}$ that corresponds to the equation $\bar{x}_3=x_3 \oplus 1$.

As already mentioned, the function given in SOP *f* or STF Y^1, that is a subject to the conversion of FPRM polynomial of a *C*-polarity is to be at first orthogonalized. Let us consider conversion of "STF $Y^1 \overset{C}{\Rightarrow}$ PSTF $Y^{\oplus}$" on the example SOP of the function $f=x_1\bar{x}_2\bar{x}_3 \vee x_2\bar{x}_3 \vee x_3$ that has not orthogonal conjuncterms. Let for this function find Zhehalkin polynomial and FPRM polynomial of (010)-polarity. For this we write it so: STF $Y^1=\{(1-0),(-0-),(--1)\}^1$ and perform orthogonalization procedure (operator $\overset{ort}{\Rightarrow}$) by set-theoretical method [9, 11], which for this function consists of two steps:

step 1: $\{(1-0),(-0-)\}\overset{ort}{\Rightarrow}\{\left((1-0)\cap\overline{(-0-)}\right),(-0-)\}=\{\left((1-0)\cap(-1-)\right),(-0-)\}=\{(110),(-0-)\}$.

In the step 1 of the procedure we obtain the set $\{(110),(-0-);(--1)\}$;

step 2: $\{(-0-),(--1)\}\overset{ort}{\Rightarrow}\{\left((-0-)\cap\overline{(--1)}\right),(--1)\}=\{\left((-0-)\cap(--0)\right),(--1)\}=\{(-00),(--1)\}$.

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So orthogonalized function *f* reflects the STF $Y^1 = \{(110), (-00), (--1)\}$.

As a result of conversion of obtained STF Y^1 into numeric Zhegalkin polynomial we have:

$$Y^1 = \{(110), (-00), (--1)\}^1 \overset{111}{\Rightarrow} \{ \begin{pmatrix} 11\mathbf{1} \\ 11- \end{pmatrix}, \begin{pmatrix} \mathbf{-11} \\ -1- \\ \cancel{--1} \\ --- \end{pmatrix}, \cancel{(--1)} \}^{\oplus} \Rightarrow \{(111), (11-), (-11), (-1-), (---)\}^{\oplus},$$

To the obtained numeric Zhegalkin polynomial corresponds the analytical expression:

$$f = x_1 x_2 \bar{x}_3 \vee \bar{x}_2 \bar{x}_3 \vee x_3 = x_1 x_2 \bar{x}_3 \oplus \bar{x}_2 \bar{x}_3 \oplus x_3 = x_1 \bar{x}_2 (x_3 \oplus 1) \oplus (x_2 \oplus 1)(x_3 \oplus 1) \oplus x_3 =$$
$$= x_1 x_2 x_3 \oplus x_1 x_2 \oplus x_2 x_3 \oplus x_2 \oplus 1.$$

As a result of conversion STF Y^1 into numeric FPRM polynomial with (010)-polarity we obtain:

$$Y^1 = \{(110), (-00), (--1)\}^1 \overset{010}{\Rightarrow} \{ \begin{pmatrix} \mathbf{0}10 \\ \cancel{-10} \end{pmatrix}, \begin{pmatrix} \cancel{-\mathbf{1}0} \\ \cancel{--0} \end{pmatrix}, \begin{pmatrix} \cancel{--\mathbf{0}} \\ --- \end{pmatrix} \}^{\oplus} \Rightarrow \{(010), (---)\}^{\oplus}.$$

To the obtained numeric FPRM polynomial with (010)-polarity corresponds the analytical expression:

$$f = x_1 x_2 \bar{x}_3 \vee \bar{x}_2 \bar{x}_3 \vee x_3 = (\bar{x}_1 \oplus 1) x_2 \bar{x}_3 \oplus (x_2 \oplus 1) \bar{x}_3 \oplus \bar{x}_3 \oplus 1 = \bar{x}_1 x_2 \bar{x}_3 \oplus 1.$$

We show that by the proposed method is easier to implement analytical transformation to go from SOP into polynomial of given *C*-polarity if compared to the "traditional" RM-expansion method of function *f* on the basis of expressions $\bar{x} = x \oplus 1$ and/or $x = \bar{x} \oplus 1$. This transformation of expression of conjuncterms of function *f*, which is given in SOP, is performed according to the algorithm described above. On the step 1 we write the expression that reflects the given polarity code *C* for significant literals of forming conjuncterms by the rule of writing (*i*-th position code → literal): $0_i \to \bar{x}_i$, $1_i \to x_i$. On the step 2 and subsequent steps, instead of change operation of positions for the dashes (–) the literals of first expression that distinguish it from the forming conjuncterm are successively eliminated. Since the transformation is performed in polynomial format, pairs of identical expressions of formed conjuncterms are eliminated from consideration. We show this in the considered above example, marking the literals of first expression that are to be eliminated by bold font:

$$f = x_1 x_2 \bar{x}_3 \vee \bar{x}_2 \bar{x}_3 \vee x_3 \overset{111}{\Rightarrow} \{ \begin{pmatrix} x_1 x_2 \mathbf{x_3} \\ x_1 x_2 \end{pmatrix}, \begin{pmatrix} \mathbf{x_2 x_3} \\ x_2 \\ \cancel{x_3} \\ 1 \end{pmatrix}, \cancel{(x_3)} \}^{\oplus} = x_1 x_2 x_3 \oplus x_1 x_2 \oplus x_2 x_3 \oplus x_2 \oplus 1;$$

$$f = x_1 x_2 \bar{x}_3 \vee \bar{x}_2 \bar{x}_3 \vee x_3 \overset{010}{\Rightarrow} \{ \begin{pmatrix} \mathbf{\bar{x}_1} x_2 \bar{x}_3 \\ \cancel{x_2 \bar{x}_3} \end{pmatrix}, \begin{pmatrix} \cancel{\mathbf{x_2} \bar{x}_3} \\ \cancel{\bar{x}_3} \end{pmatrix}, \begin{pmatrix} \cancel{\mathbf{\bar{x}_3}} \\ 1 \end{pmatrix} \}^{\oplus} = \bar{x}_1 x_2 \bar{x}_3 \oplus 1.$$

On the basis of the described method one can perform arbitrary conversions from one *C*-polarity to another. The following example illustrates this.

Example. For function *f* given by canonical STF $Y^1 = \{2,7,9,12,15\}^1$ one can find by set-theoretical method Zhegalkin polynomial and FPRM polynomials with (1110)-polarity and (1010)-polarity[1].

[1] This function is taken from [5] where this example is solved by Karnaugh maps method.

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Solution. $Y^1=\{(0010),(0111),(1001),(1100),(1111)\}^1 \overset{1111}{\Rightarrow}$

$$\overset{1111}{\Rightarrow}\left\{\begin{pmatrix}\cancel{\mathbf{1111}}\\ \cancel{111-}\\ \cancel{1-11}\\ \cancel{-111}\\ 1-1-\\ -11-\\ --11\\ --1-\end{pmatrix},\begin{pmatrix}\cancel{\mathbf{1}111}\\ \cancel{-111}\end{pmatrix},\begin{pmatrix}\cancel{1\mathbf{11}1}\\ \cancel{11-1}\\ \cancel{1-11}\\ 1--1\end{pmatrix},\begin{pmatrix}\cancel{11\mathbf{11}}\\ \cancel{111-}\\ \cancel{11-1}\\ 11--\end{pmatrix},(1111)\right\}^{\oplus}\Rightarrow\left\{\begin{pmatrix}1-1-\\ -11-\\ --11\\ --1-\end{pmatrix},(1--1),(11--),(1111)\right\}^{\oplus}.$$

The obtained numeric Zhegalkin polynomial will be transformed into its analytic equivalent:

$$f(x_1,x_2,x_3,x_4)=x_1x_3\oplus x_2x_3\oplus x_3x_4\oplus x_3\oplus x_1x_4\oplus x_1x_2\oplus x_1x_2x_3x_4.$$

To obtain FPRM polynomial with (1110)-polarity we will do the conversion procedure only for the elements of numeric Zhegalkin polynomial that have value 1 in the position with weight 2^0:

$$(--11)\overset{1110}{\Rightarrow}\begin{pmatrix}--1\mathbf{0}\\ --1-\end{pmatrix},\quad (1--1)\overset{1110}{\Rightarrow}\begin{pmatrix}1--\mathbf{0}\\ 1---\end{pmatrix},\quad (1111)\overset{1110}{\Rightarrow}\begin{pmatrix}111\mathbf{0}\\ 111-\end{pmatrix}.$$

Having changed by these sets the corresponding elements of numeric Zhegalkin polynomial and having done simplification procedure, we will get FPRM polynomial with (1110)-polarity:

$$\left\{\begin{pmatrix}1-1-\\ -11-\\ \cancel{--10}\\ \cancel{--1-}\\ --1-\end{pmatrix},\begin{pmatrix}1--0\\ 1---\end{pmatrix},(11--),\begin{pmatrix}1110\\ 111-\end{pmatrix}\right\}^{\oplus}\Rightarrow x_1x_3\oplus x_2x_3\oplus x_3\bar{x}_4\oplus x_1\bar{x}_4\oplus x_1\oplus x_1x_2\oplus x_1x_2x_3\bar{x}_4\oplus x_1x_2x_3.$$

As a result of direct transformation of canonical STF Y^1 we will get analogical result:

$$Y^1=\{(0010),(0111),(1001),(1100),(1111)\}^1\overset{1110}{\Rightarrow}\left\{\begin{pmatrix}\cancel{\mathbf{11}10}\\ \cancel{1-10}\\ \cancel{-110}\\ --10\end{pmatrix},\begin{pmatrix}\cancel{\mathbf{1}11\mathbf{0}}\\ \cancel{111-}\\ \cancel{-110}\\ -11-\end{pmatrix},\begin{pmatrix}\cancel{1\mathbf{110}}\\ \cancel{111-}\\ \cancel{11-0}\\ \cancel{1-10}\\ 11--\\ 1-1-\\ 1--0\\ 1---\end{pmatrix},\begin{pmatrix}\cancel{11\mathbf{1}0}\\ \cancel{11-0}\end{pmatrix},\begin{pmatrix}111\mathbf{0}\\ 111-\end{pmatrix}\right\}^{\oplus}\Rightarrow$$

$$\Rightarrow\left\{(--10),(-11-),\begin{pmatrix}11--\\ 1-1-\\ 1--0\\ 1---\end{pmatrix},\begin{pmatrix}1110\\ 111-\end{pmatrix}\right\}^{\oplus}.$$

FPRM polynomial with (1010)-polarity can now be obtained in several ways: either from numeric Zhegalkin polynomial, or from numeric FPRM polynomial with (1110)-polarity, or by direct conversion of the given function *f*. It is simpler to find the searched polynomial on the basis of already obtained numeric

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FPRM polynomial with (1110)-polarity having done the conversion procedure with those its elements that have 1 in the position with weight 2^2:

$$(-11-) \overset{1010}{\Rightarrow} \begin{pmatrix} -\mathbf{0}1- \\ --1- \end{pmatrix}, \quad (11--) \overset{1010}{\Rightarrow} \begin{pmatrix} 1\mathbf{0}-- \\ 1--- \end{pmatrix}, \quad (1110) \overset{1010}{\Rightarrow} \begin{pmatrix} 1\mathbf{0}10 \\ 1-10 \end{pmatrix}, \quad (111-) \overset{1010}{\Rightarrow} \begin{pmatrix} 1\mathbf{0}1- \\ 1-1- \end{pmatrix}.$$

After doing the appropriate substitution and simplifying procedure we get:

$$\{(--10), \begin{pmatrix} -01- \\ --1- \end{pmatrix}, \begin{pmatrix} 10-- \\ \cancel{1---} \\ \cancel{1-1-} \\ 1--0 \\ \cancel{1---} \end{pmatrix}, \begin{pmatrix} 1010 \\ 1-10 \\ 101- \\ \cancel{1-1-} \end{pmatrix}\}^{\oplus} \Rightarrow \{(--10), \begin{pmatrix} -01- \\ --1- \end{pmatrix}, \begin{pmatrix} 10-- \\ 1--0 \end{pmatrix}, \begin{pmatrix} 1010 \\ 1-10 \\ 101- \end{pmatrix}\}^{\oplus}.$$

So, FPRM polynomial with (1010)-polarity of function

$$f(x_1, \bar{x}_2, x_3, \bar{x}_4) = x_3\bar{x}_4 \oplus \bar{x}_2 x_3 \oplus x_3 \oplus x_1\bar{x}_2 \oplus x_1\bar{x}_4 \oplus x_1\bar{x}_2 x_3\bar{x}_4 \oplus x_1 x_3\bar{x}_4 \oplus x_1\bar{x}_2 x_3.$$

If the given function *is* immediately transformed into numeric FPRM polynomial with (1010)-polarity, the result will be the same:

$$Y^1 = \{(0010),(0111),(1001),(1100),(1111)\}^1 \overset{1010}{\Rightarrow} \{\begin{pmatrix} \cancel{\mathbf{1}010} \\ \cancel{-010} \end{pmatrix}, \begin{pmatrix} \cancel{1\mathbf{0}1\mathbf{0}} \\ \cancel{101-} \\ \cancel{1-10} \\ \cancel{-010} \\ \cancel{1-1-} \\ -01- \\ --10 \\ --1- \end{pmatrix}, \begin{pmatrix} \cancel{1\mathbf{0}1\mathbf{0}} \\ \cancel{101-} \\ \cancel{10-0} \\ 10-- \end{pmatrix}, \begin{pmatrix} \cancel{1\mathbf{01}0} \\ \cancel{10-0} \\ \cancel{1-10} \\ 1--0 \end{pmatrix}, \begin{pmatrix} 1\mathbf{0}1\mathbf{0} \\ 101- \\ 1-10 \\ \cancel{1-1-} \end{pmatrix}\}^{\oplus} \Rightarrow$$

$$\Rightarrow \{\begin{pmatrix} -01- \\ --10 \\ --1- \end{pmatrix}, (10--), (1--0), \begin{pmatrix} 1010 \\ 101- \\ 1-10 \end{pmatrix}\}^{\oplus}.$$

Conclusions

The described numeric set-theoretical method of obtaining of given polarity variables polynomials of logic functions of *n* variables differs from known methods by simpler implementation and possibility of direct application of it on the computer without any intermediate transformations. The method can also be applied to the system of logic functions of *n* variables of arbitrary forms of its giving.

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