KINEMATIC ANALYSIS OF CENTRIFUGAL
AND AERODYNAMIC MECHANISMS OF BLADES TURNING
AND FOLDING OF HORIZONTAL-AXIS WIND TURBINES

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Problem stating. During the kinematic analysis the mechanism parts motion is being investigated without taking into consideration actuating loads. In other words the motion of its parts is being considered from the geometrical point of view taking into account only time factor [1]. Determining of positions, displacements, velocities and accelerations of separate mechanism parts and constructing of their motion paths are the main problems of kinematic analysis. The accordance of the mechanism kinematic parameters to its preplanned operating conditions and the basic data for further dynamic investigations are being considered as a result of kinematic analysis [1]. Also kinematic analysis results may be used for solving such problems as synthesis and optimization of mechanisms constructions.

Analytical methods are reasonably used for kinematic analysis of the mechanism under conditions in which its parts move according to some preplanned laws [1]. These methods allow to put into practice multivariate investigations of mechanisms and to realize optimization synthesis of their constructions. A quantity of transactions concerned with analytical investigations of link mechanisms are published in technical literature. Among universal methods we may distinguish two basic types: 1) the method of vector lopped circuits introduced by V. A. Zinovyev; 2) the method of coordinates transformation developed by Yu. F. Moroshkin [1]. The first method is more convenient for kinematic analysis of planar mechanisms, the second method is expedient for spatial mechanisms analysis.

The systems of blades turning and folding of horizontal-axis wind turbines are used for power regulation and rotation frequency stabilization under conditions of changeable wind speed and loading on wind-wheel driving shaft [2]. Some of their constructions may be presented as planar or as spatial mechanisms. That’s why both mentioned methods of kinematic analysis will be used in further research. The importance of these investigations is caused by necessity of constructions optimization of horizontal-
axis wind turbines of various types (high-speed, low-speed multiblade etc.) with the purpose of their production costs reduction and improvement of reliability and efficiency.

**Analysis of recent investigations and publications.** Regulation mechanisms are used in most modern wind turbines of nominal power from 1-2 kW to several MW. Active regulation systems, in which blades are actuated with the help of electric or hydraulic drives, are mostly used in wind turbines of middle and high power (over 50 kW). At the same time so-called “small” wind turbines (under 10-20 kW) are usually equipped by mechanical systems of aerodynamic, centrifugal or combined centrifugal and aerodynamic types [2; 3; 4].

Most recent scientific investigations and developments are aimed at improvement of serial wind turbines of middle and high power, which are predominately used for electrical energy producing. At the same time “small” wind energetics develops at essentially slower pace and problems of improvement of mechanical systems of power regulation of “small” wind turbines are disregarded in increasing frequency [5]. Among recent internal investigations of regulation mechanisms we may find transactions of such scientists as S. O. Kudrya, V. M. Holovko, V. P. Kohanyevych [6; 7], V. S. Kryvtsov, O. M. Oleynikov, O. I. Yakovlyev [8], D. V. Legochyn [9] etc. Foreign advance at improving of wind turbines power regulation systems are presented by numerous developments of American and European researchers: Bourriaud Michel Edouard Raymond (France), Roland Stoer (Germany), Curt Higgins, Mark Folsom, John Pitre, Stuart Huang, Kenneth James Deering, Peter Mok (USA) [10; 11] etc.

Most analyzed publications contain different information about constructing of mathematical models of wind turbine aeromechanical system and about investigating of its operation features at typical transitional modes. In most cases recent investigations don’t analyze the influence of kinematic parameters on processes of power regulation or rotation frequency stabilization. At that existent regulation mechanisms can’t be always considered as optimal ones with relation to their construction. Improving of existent and synthesis of new mechanisms provides not only for laying of construction idea and functioning hypothesis, but also for its kinematic and dynamic analyzing with the purpose of establishing the accordance of mechanism parameters to its preplanned operating conditions.

**Formulating of article purpose.** Analyzing of existent mechanical systems of power regulation and rotation frequency stabilization of horizontal-axis wind turbines, which function at the expense of blades turning and folding. Constructing of simplified kinematic diagrams of corresponding mechanisms. Deducing of analytical dependencies for calculating of kinematic parameters (displacements, velocities and accelerations) of separate parts of various mechanisms with the purpose of further establishing the accordance of the mechanisms parameters to their preplanned operating conditions and of determining of output parameters for dynamic analysis (calculating of dynamic loads, kinetic energy, mechanical power etc.).

**Stating of basic results.** Blades turning and folding mechanisms are the most effective methods of power regulation of horizontal-axis wind turbines of low power [2; 3]. When using the first mechanism wind-wheel blades can rotate round their own longitudinal axes (fig. 1, a, c) and change their attack angles with respect to incoming air-flow direction. As a result of blades turning the character of blades streamline by incoming air-flow changes. The effect of flow stalling appears and causes decreasing of lifting force and wind turbine power [2; 3; 4]. The second mechanism operate by the principle of “umbrella” and enables blades turning in direction, which is perpendicular to wind-wheel rotation plane. Blades can fold along wind-wheel shaft (fig. 1, b, d) decreasing of its blow-off area and reducing of power takeoff [2; 3].

Both methods may use the effect of changing of air-flow ram pressure on blades surfaces as a result of increasing or decreasing of wind speed [2; 3]. This effect leads to blades angular position changing. In most cases the mechanisms of blades turning and folding are equipped by auxiliary centrifugal regulator, which represents the system of weights. These weights can change their position with respect to wind-wheel rotation axis. In that case wind turbine rotation frequency exerts the main control influence on blades position and on the process of power regulation [2; 3]. The mechanisms of blades turning and folding, which use the effect of ram pressure changing on blades surfaces and centrifugal regulator, are called combined centrifugal and aerodynamic regulation mechanisms.
Combined centrifugal and aerodynamic mechanisms of blades turning and folding are the most interesting from the point of view of kinematic analysis because of large variety of positions and motion paths of regulation weights. With the purpose of maximal regulation accuracy achieving and construction mass minimizing it is necessary to find the most optimal positions and motion paths of regulation weights. Four different versions of centrifugal weights positioning will be analyzed in this article. Planar kinematic diagrams of different mechanisms of blades turning and folding are presented at fig. 2.

In further investigations we will analyze the mechanisms of blades turning and folding with one degree of freedom. At that the initial link (the rod \( OD \) (fig. 2, a-d) or blade axis \( DV \) (fig. 2, e-h)) of all mechanisms accomplishes rotary axis motion. That's why the corresponding angle \( \phi \) is chosen as the generalized coordinate, which definitely determines the position of the rest units of the mechanism. The position of the coordinates origin is placed in the point \( O \), the abscissa axis is directed to the right along wind-wheel axis and the ordinate axis is directed vertically upwards. The dimensions of the units and the angles of their mutual bracing are the output data for kinematic analysis.

The coordinates of the point \( D \) which belongs to the crank \( OD \) of blades turning mechanisms (fig. 2, a-d) can be found from the following equation:

\[
x_D = l_{OD} \cdot \sin(\phi + \phi_{OD}); \quad y_D = l_{OD} \cdot \cos(\phi + \phi_{OD}),
\]

where \( \phi \) – the angle between the blade chord and wind-wheel rotation plane, which is perpendicular to its shaft; \( \phi_{OD} \) – the angle between the rod \( OD \) and the blade chord; \( l_{OD} \) – the length of the rod \( OD \).
The ordinate of the point \( N \) is constant. At the same time its horizontal displacement is equal to the sum of two items: the displacement of the point \( D \) along the abscissa axis – \( x_D \) and horizontal displacement \( x_{DN} \), which is generated due to turning of the rod \( DN \) round the hinge \( N \) (fig. 2, a, b, d):

\[
x_N = x_D + \sqrt{l_{DN}^2 - (y_D - y_N)^2}; \quad y_N = l_{NH} \cdot \sin(\phi_{NH}).
\]  

(2)

where \( l_{DN}, l_{NH} \) – the length of the rods \( DN \) and \( NH \); \( \phi_{NH} \) – the angle of tilt of the rod \( NH \) about the wind-wheel shaft.

When substituting of the expressions (1) into the formulas (2) the horizontal coordinate of the slider \( H \) (fig. 2, a, b, d) can be found from the equation:

\[
x_H = l_{OD} \cdot \sin(\phi + \phi_{OD}) + \sqrt{l_{DN}^2 - (l_{OD} \cdot \cos(\phi + \phi_{OD}) - l_{NH} \cdot \sin(\phi_{NH}))^2} - l_{NH} \cdot \cos(\phi_{NH}).
\]

(3)

When differentiating of the equation (3) with respect to the generalized coordinate \( \phi \) the analogue of velocity of the slider \( H \) of blades turning mechanisms (fig. 2, a, b, d) can be determined. As a result of double differentiating of the equation (3) the analogue of acceleration of the slider \( H \) can be determined.

For determining of kinematic parameters of the slider \( H \) of blades folding mechanisms (fig. 2, e, f, h) it is necessary to find the coordinates of the point \( E \), which belongs to the crank \( OVE \):

\[
x_E = l_{OD} \cdot \cos(\phi_{OD}) + \sqrt{l_{DV}^2 + l_{VE}^2 + 2 \cdot l_{DV} \cdot l_{VE} \cdot \cos(\phi_{VE})} \times
\]

\[
\times \sin \left( \arcsin \left( \frac{l_{VE} \cdot \sin(\phi_{VE})}{\sqrt{l_{DV}^2 + l_{VE}^2 + 2 \cdot l_{DV} \cdot l_{VE} \cdot \cos(\phi_{VE})}} + \phi \right) \right);
\]

\[
y_E = l_{OD} \cdot \sin(\phi_{OD}) + \sqrt{l_{DV}^2 + l_{VE}^2 + 2 \cdot l_{DV} \cdot l_{VE} \cdot \cos(\phi_{VE})} \times
\]

\[
\times \cos \left( \arcsin \left( \frac{l_{VE} \cdot \sin(\phi_{VE})}{\sqrt{l_{DV}^2 + l_{VE}^2 + 2 \cdot l_{DV} \cdot l_{VE} \cdot \cos(\phi_{VE})}} + \phi \right) \right),
\]

(4)

where \( \phi \) – the angle between blade longitudinal axis and wind-wheel rotation plane, which is perpendicular to its shaft; \( \phi_{VE} \) – the angle between the rod \( VE \) and blade longitudinal axis; \( l_{OD}, l_{DV}, l_{VE} \) – the length of the rods \( OD, DV \) and \( VE \).

The ordinate of point \( N \) is constant. At the same time its horizontal displacement is equal to the sum of two items: the displacement of the point \( E \) along the abscissa axis – \( x_E \) and horizontal displacement \( x_{EN} \), which is generated due to turning of the rod \( EN \) round the hinge \( N \) (fig. 2, e, f, h):

\[
x_N = x_E + \sqrt{l_{EN}^2 - (y_E - y_N)^2}; \quad y_N = l_{NH} \cdot \sin(\phi_{NH}).
\]

(5)

where \( l_{EN} \) – the length of the rod \( EN \).

When substituting of the expressions (5) into formulas (4) the horizontal coordinate of slider \( H \) (fig. 2, e, f, h) can be found from the equation:

\[
x_H = l_{OD} \cdot \cos(\phi_{OD}) + \sqrt{l_{DV}^2 + l_{VE}^2 + 2 \cdot l_{DV} \cdot l_{VE} \cdot \cos(\phi_{VE})} \times
\]

\[
\times \sin \left( \arcsin \left( \frac{l_{VE} \cdot \sin(\phi_{VE})}{\sqrt{l_{DV}^2 + l_{VE}^2 + 2 \cdot l_{DV} \cdot l_{VE} \cdot \cos(\phi_{VE})}} + \phi \right) \right) +
\]

(6)
\[
\left(1 + \frac{l_{EN}^2}{l_{EN}^2 - \left[\frac{l_{OD} \cdot \sin(\phi_{OD}) + \sqrt{l_{DV}^2 + l_{VE}^2 + 2 \cdot l_{DV} \cdot l_{VE} \cdot \cos(\phi_{VE})}}{\sqrt{l_{DV}^2 + l_{VE}^2 + 2 \cdot l_{DV} \cdot l_{VE} \cdot \cos(\phi_{VE})}}\right]^2}
\right) - l_{NH} \cdot \cos(\phi_{NH}).
\]

Fig. 2. Planar kinematic diagrams of various types of combined centrifugal and aerodynamic mechanisms of blades turning (a, b, c, d) and folding (e, f, g, h).
When differentiating of the equation (6) with respect to the generalized coordinate ϕ the analogue of velocity of the slider H (fig. 2, e, f, h) can be determined. As a result of double differentiating of the equation (6) the analogue of acceleration of the slider H can be determined.

The intercoupling between the blade (or the centrifugal weight) turning and the regulation slider displacing for mechanisms, which are shown at fig. 2, a, e, may be definitely described using equations (3) and (6). When using of blades turning and folding mechanisms, which are presented at fig. 2, b, f, the rectilinear displacement of the slider V (with the weight A) depends on horizontal displacement of the slider H and may be determined from the following relation:

\[ l_{NV}^2 = \left[ l_{OM} + \frac{l_{HN} \cdot \sin(\phi_{NH})}{\tan(\phi_{VM})} \right] - x_N^2 + \left[ l_{MV} - \frac{l_{HN} \cdot \sin(\phi_{NH})}{\sin(\phi_{VM})} \right]^2 + 2 \cdot \left[ l_{OM} + \frac{l_{HN} \cdot \sin(\phi_{NH})}{\tan(\phi_{VM})} \right] - x_N \cdot \left[ l_{MV} - \frac{l_{HN} \cdot \sin(\phi_{NH})}{\sin(\phi_{VM})} \right] \cdot \cos(\phi_{VM}), \]

where \( l_{NV} \) – the length of the rod NV; \( l_{OM} \) – the distance between the points O and M; \( l_{MV} \) – the distance between the point M and the hinge V; \( \phi_{VM} \) – the angle between the rod VM and wind-wheel axis; \( x_N \) – the horizontal displacement of the hinge N, which may be determined using of formulas (2) (for blades turning mechanism (fig. 2, b)) or formulas (5) (for blades folding mechanism (fig. 2, f)).

When solving of the equation (7) we have found two roots. One of them has negative value. That’s why this root may be neglected. Thus the distance between wind-wheel axis and the slider V (with the weight A) when moving along the rod VM (fig. 2, b, f) may be determined from the formula:

\[ l_{VM} = \sqrt{l_{NV}^2 - \left[ l_{OM} + \frac{l_{HN} \cdot \sin(\phi_{NH})}{\tan(\phi_{VM})} \right] - x_N} \cdot \sin^2(\phi_{VM}) - \left[ l_{OM} + \frac{l_{HN} \cdot \sin(\phi_{NH})}{\tan(\phi_{VM})} \right] - x_N \cdot \cos(\phi_{VM}). \]

Using of the formula (8) we may deduce the equation for determining of horizontal and vertical displacement of the weight A (fig. 2, b, f):

\[ l_{OAx} = l_{OM} + l_{VM} \cdot \cos(\phi_{VM}) - l_{VA} \cdot \cos(\phi_{VA} + \phi_{VM}); \]
\[ l_{OAy} = l_{VM} \cdot \sin(\phi_{VM}) - l_{VA} \cdot \sin(\phi_{VA} + \phi_{VM}). \]

It is necessary to substitute corresponding expressions for determining of the displacement \( x_N \) (for blades turning mechanism (fig. 2, b) – the expressions (1) and (2); for blades folding mechanism (fig. 2, f) – the expressions (4) and (5)) into the equation (8). When differentiating of the equation (8) with respect to the generalized coordinate ϕ the analogue of velocity of the weight A (and the slider V) of blades turning and folding mechanisms (fig. 2, b, f) can be determined. As a result of double differentiating of the equation (8) the analogue of acceleration of the weight A (and the slider V) can be determined.

When using of blades turning and folding mechanisms, which are presented at fig. 2, d, h, the rectilinear displacement of the slider V depends on horizontal displacement of the sliders H and M. For determining of horizontal \( x_V \) and vertical \( y_V \) positions of the slider V with respect to the coordinates origin (point O) the following system of two equations with two unknowns \( \phi_{NV} \) i \( \phi_{VK} \) may be used:

\[ y_V = l_{HN} \cdot \sin(\phi_{HN}) + l_{NV} \cdot \sin(\phi_{NV}) = l_{VK} \cdot \sin(\phi_{VK}); \]
\[ x_V = x_H + l_{HN} \cdot \cos(\phi_{HN}) + l_{NV} \cdot \cos(\phi_{NV}) = l_{OK} - l_{VK} \cdot \cos(\phi_{VK}). \]
where $\varphi_{NV}$ and $\varphi_{VK}$ – the unknown angles between the rods $NV$, $VK$ and wind-wheel rotation axis; $l_{OK}$ – the distance between the coordinates origin (point $O$) and the hinge $K$; $l_{VK}$ – the length of the rod $VK$; $x_H$ – the displacement of the slider $H$, which may be calculated using formulas (3) (for blades turning mechanisms (fig. 2, d)) or formulas (6) (for blades folding mechanism (fig. 2, h)).

The solutions of the system (10) (especially the expressions for determining of $\varphi_{NV}$), found with the help of programmatic complex MathCAD, are of the form:

$$\varphi_{NV} = \arctan \left[ \frac{\left( l_{NH} \cdot \sin(\varphi_{NH}) \right)^2 + \left( l_{NV} \right)^2 - \left( l_{VK} \right)^2 }{2 \cdot l_{NV} \cdot \left( l_{OK} - x_H - l_{NH} \cdot \cos(\varphi_{NH}) \right) } \right] \pm \frac{1}{2} \cdot \frac{l_{NH} \cdot \sin(\varphi_{NH})}{\sqrt{\left( l_{OK} - x_H - l_{NH} \cdot \cos(\varphi_{NH}) \right)^2 + \left( l_{NV} \right)^2 - \left( l_{VK} \right)^2}}$$

When substituting of the equation (11) into the expressions (10) the coordinates $x_V$ and $y_V$ of the slider $V$ (fig. 2, d, h) may be found. On basis of the formulas (10) and (11) the equations for determining of horizontal and vertical displacements of the weight $A$ may be deduced:

$$\begin{cases} y_A = y_V + l_{VA} \cdot \sin(\varphi_{VM} - \varphi_{VA}) = l_{HN} \cdot \sin(\varphi_{HN}) + l_{NV} \cdot \sin(\varphi_{NV}) ; \\ x_A = x_V + l_{VA} \cdot \cos(\varphi_{VM} - \varphi_{VA}) = x_H + l_{HN} \cdot \cos(\varphi_{HN}) + l_{NV} \cdot \cos(\varphi_{NV}) + l_{VA} \cdot \cos(\varphi_{VM} - \varphi_{VA}) \end{cases}$$

(12)

It is necessary to substitute corresponding expressions for determining of the displacement $x_H$ (for blades turning mechanisms (fig. 2, d) – the expression (3); for blades folding mechanism (fig. 2, h) – the expression (6)) into the equation (12). When differentiating of the equation (12) with respect to the generalized coordinate $\varphi$ the analogue of velocity of the weight $A$ (and the slider $V$) of blades turning and folding mechanisms (fig. 2, d, h) can be determined. As a result of double differentiating of the equation (12) the analogue of acceleration of the weight $A$ (and the slider $V$) can be determined.
Horizontal displacement of the slider $M$ of blades turning and folding mechanisms (fig. 2, d, h) may be found from the formula: $x_M = x_V - l_{VM} \cdot \cos(\varphi_{VM})$. Kinematic parameters (analogues of velocity and acceleration) of the slider $M$ are equal to corresponding kinematic parameters of horizontal movement of the slider $V: \frac{dx_V}{d\varphi} \cdot d\varphi$ and $\frac{d^2x_V}{d\varphi^2}$.

In certain cases the centrifugal weights may be placed on auxiliary steering axes attached at an angle to the wind-wheel shaft and mounted between the hub and the regulation slider. Kinematic schemes of corresponding regulation mechanisms are presented at fig. 2, c, g. When using of blades turning mechanism (fig. 2, c) the coordinates of the slider with centrifugal weight are equal to the coordinates of the point $D$ and may be found from the formula (1). Then horizontal position of the sliders $M$ and $H$ may be determined from the following expressions:

$$x_M = x_D - \frac{y_D}{\tan(\varphi_{MD})};$$

$$x_H = x_D + \sqrt{(l_{DN})^2 - (y_D - l_{NH} \cdot \sin(\varphi_{NH}))^2} - l_{NH} \cdot \cos(\varphi_{NH}).$$

When using of blades folding mechanism, shown at fig. 2, g, it is necessary to find the coordinates of the hinge $V$ firstly:

$$x_V = l_{OD} \cdot \cos(\varphi_{OD}) + l_{DV} \cdot \sin(\varphi);$$

$$y_V = l_{OD} \cdot \sin(\varphi_{OD}) + l_{DV} \cdot \cos(\varphi).$$

For determining of horizontal $x_E$ and vertical $y_E$ positions of the slider $E$ (fig. 2, g) with respect to the coordinates origin (point $O$) the following system of two equations may be used:

$$\begin{cases}
    x_E = l_{OM} + \frac{y_E}{\tan(\varphi_{ME})}; \\
    y_E = y_V + \sqrt{(l_{VE})^2 - (x_E - x_V)^2},
\end{cases}$$

where $\varphi_{ME}$ – the angle between the rod $ME$ and the wind-wheel shaft (fig. 2, g).

The solutions of the system (15), found with the help of MathCAD, are of the form:

$$x_E = \frac{x_V \cdot \tan(\varphi_{ME}) + l_{OM} \cdot \tan^2(\varphi_{ME})}{\sqrt{(l_{VE})^2 \cdot (\tan^2(\varphi_{ME}) + 1) - (l_{OM} \cdot \tan(\varphi_{ME}) + y_V)^2}} + \frac{\sqrt{(l_{VE})^2 \cdot (\tan^2(\varphi_{ME}) + 1) - (l_{OM} \cdot \tan(\varphi_{ME}) + y_V)^2}}{\tan^2(\varphi_{ME}) + 1};$$

$$y_E = \tan(\varphi_{ME}) \cdot \frac{x_V \cdot l_{OM} + y_V \cdot \tan(\varphi_{ME})}{\sqrt{(l_{VE})^2 \cdot (\tan^2(\varphi_{ME}) + 1) - (l_{OM} \cdot \tan(\varphi_{ME}) + y_V)^2}} + \frac{\sqrt{(l_{VE})^2 \cdot (\tan^2(\varphi_{ME}) + 1) - (l_{OM} \cdot \tan(\varphi_{ME}) + y_V)^2}}{\tan^2(\varphi_{ME}) + 1}.$$  \( \text{(16)} \)

On basis of formulas (16) the positions of the hinge $N$ and the slider $H$ (fig. 2, g) may be found:

$$x_N = x_E + \sqrt{(l_{EN})^2 - (y_E - l_{NH} \cdot \sin(\varphi_{NH}))^2}; \quad x_H = x_N - l_{NH} \cdot \cos(\varphi_{NH}).$$  \( \text{(17)} \)

Kinematic parameters (analogues of velocities and accelerations) of separate parts of blades turning and folding mechanisms, shown at fig. 2, c, g, may be determined by single and double differentiating of the corresponding equations (13-17) with respect to the generalized coordinate $\varphi$.

**Conclusions.** Various types of blades turning and folding mechanisms, which may be used for power regulation and rotation frequency stabilization of horizontal-axis wind turbines, are considered. Kinematic diagrams of corresponding mechanisms are constructed (fig. 2). Analytical expressions for
calculating of regulation slider and centrifugal weights displacements as functions of blades turning angle are deduced (1-17). The recommendations for determining of their kinematic parameters (analogues of velocities and accelerations) are presented with the purpose of further establishing the accordance of the mechanisms parameters to their preplanned operating conditions and of determining of output parameters for dynamic analysis (calculating of dynamic loads, kinetic energy, mechanical power etc.).

In further investigations it is planned to make up the computer modelling of motion of separate parts for dynamic analysis (calculating of dynamic loads, kinetic energy, mechanical power etc.).

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