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FOURIER METHOD EMPLOYED FOR SOLVING THERMOELASTICITY PROBLEMS FOR RECTANGULAR PLATE WITH OPENING AND INCLUSIONS

Dynamic problems of thermoelasticity for the rectangular plate with absolutely rigid inclusion and opening are considered. The problem is reduced to the system of boundary integral equations, numerical solution of which is constructed by the collocation method.
Key words: system of boundary integral equations, collocation method.

Розглянуто динамічні задачі термопружності для прямокутної пластини з повністю жорсткими включеннями та відкриттям. Задача зводиться до системи граничних інтегральних рівнянь, чисельні розв'язання яких будуються за допомогою методу колокації.
Ключові слова: система граничних інтегральних рівнянь, метод колокації.

Mathematical apparatus of the function expansion in the generalized Fourier series in accordance with trigonometrical function systems is applied in works [1,3] for the definition of integral equations solution definition and constructing numerical solutions to the problems the theory of elasticity in canonical fields.

In the given work the Fourier method used for formation of numerical solutions of quasi-dynamic tasks of thermostability for the isotropic plates with opening and extremely absolutely rigid inclusions has been developed. The components of the inert powers affecting the plate are completely neglected. The temperature and stresses appearing in the plate are the functions of two linear coordinates and time. We suggest that heat-exchange on flat boundaries between the plate and environment works according to Newton's law, the edges of plates are (partially) free from stresses, on the contact line(inclusion and plate) and on the edges of the plate the temperature is set.

Firld let's designate as $S=\{x(x,y):0<x<l_1;\quad 0<y<l_2\}$, where determined functions set thermoelastic state of plate, and L – line(ellipse) of contact of inclusion and plate or opening boundary. Parametric equation of this line is the following

$$x=l_1/2+b_1\cos J,\ \ y=l_2/2-b_2\sin J,\quad 0\le J<2p\ , \tag{1}$$

where xOy – is Decartes coordinate system, b_1, b_2 – is semi-axes of ellipse.

Isolated tangent and normal vector to the line L assume the following form

$$\overset{\mathbf{r}}{t}=\{t_1;t_2\},\ \overset{\mathbf{r}}{n}=\{n_1;n_2\}, \tag{2}$$

where $t_1=-n_2=-b_1\sin J\Big/\sqrt{b_1^2\sin^2 J+b_1^2\sin^2 J}$, $t_2=n_1=-b_2\cos J\Big/\sqrt{b_1^2\sin^2 J+b_1^2\sin^2 J}$.

Equation of the ordinary normal line at point $x(x,y)\in L$ is

$$x_r=n_1r+x,\ y_r=n_2r+y,\quad -\infty<r<\infty\text{, або }\ x_r=\overset{\mathbf{r}}{n}r+x\ . \tag{3}$$

Equations describing thermoelastic state of plate, boundary conditions on the plate edges, on the inclusion(opening) and starting conditions are the following [2]

$$a^2\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-a\,T-\frac{\partial T}{\partial t}=-g\ ,\ \frac{\partial N_{11}}{\partial x}+\frac{\partial N_{12}}{\partial y}=-p_1\ ,\ \frac{\partial N_{21}}{\partial x}+\frac{\partial N_{22}}{\partial y}=-p_2\ ,$$

$$N_{11}=B\left(\frac{\partial u_1}{\partial x}+n\,\frac{\partial u_2}{\partial y}\right)-(1+n)\,Ba_T T\ ,\ N_{22}=B\left(\frac{\partial u_2}{\partial y}+n\,\frac{\partial u_1}{\partial x}\right)-(1+n)\,Ba_T T\ ,$$

$$N_{12}=N_{21}=\frac{B(1-n)}{2}\left(\frac{\partial u_2}{\partial x}+\frac{\partial u_1}{\partial y}\right), \qquad x\in S,\ t>0; \tag{4}$$

$$T(x,t)\big|_{x=0}=0,\ T(x,t)\big|_{y=0}=0,\ T(x,t)\big|_{x=l_1}=0,\ T(x,t)\big|_{y=l_2}=0,$$

$$u_2(x,t)\big|_{x=0}=0,\ u_2(x,t)\big|_{x=l_1}=0,\ u_1(x,t)\big|_{y=0}=0,\ u_1(x,t)\big|_{y=l_2}=0,\ t\geq 0,$$

$$N_{11}(x,t)\big|_{x=0}=0,\ N_{11}(x,t)\big|_{x=l_1}=0,\ N_{22}(x,t)\big|_{y=0}=0,\ N_{22}(x,t)\big|_{y=l_2}=0; \tag{5}$$

$$T(x,t)\big|_{x\in L}=T_0(x,t),\ \ t\geq 0,\ u_1(x,t)\big|_{x\in L}=0,\ u_2(x,t)\big|_{x\in L}=0,\ \ t\geq 0, \tag{6}$$

$$(T(x,t)\big|_{x\in L}=T_0(x,t),\ \ t\geq 0,\ N_1(x,t)\big|_{x\in L}=0,\ N_2(x,t)\big|_{x\in L}=0,\ \ t\geq 0,)$$

$$T(x,t)\big|_{t=0}=0,\ x\in S, \tag{7}$$

where $N_T=B(1+n)a_T T$; $N_1=N_{11}n_1+N_{12}n_2$, $N_2=N_{21}n_1+N_2n_2$; $B=2hE\big/\left(1-n^2\right)$; $a=h_0/(cr)$; $g=f/(cr)$; $a^2=k/(cr)$; $T=t-q$ – function of temperature; t – plate temperature; $T_0(x,t)$ – function of the temperature set on the line L; q – temperature of the environment touching the plane bases of the plate; f – density of thermic souces; p_1, p_2 – are volumetric and surface powers reduced to the middle surface.

At first we form singular solution of the quasi-dynamic task of the thermoelasticity rectangle S, namely we'll find the solution to the task (4), (5) (7) (concerning the effect of the thermal source and powers gathered at point $x_0(x_0,y_0)$). Let's set the intensity outside thermal souces and powers looking like general sums of trigonometric series.

$$g=P_0(x_0,t)d(x_0,x)d(y_0,y)=P_0(x_0,t)\lim_{e\to 0}\frac{4}{l_1l_2}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}c_{km}(e)F_{km}^{ss}(x_0)F_{km}^{ss}(x),$$

$$p_1=P_1(x_0,t)d(x_0,x)d(y_0,y)=P_1(x_0,t)\lim_{e\to 0}\frac{4}{l_1l_2}\sum_{k=0}^{\infty}\sum_{m=1}^{\infty}c_{km}(e)F_{km}^{cs}(x_0)F_{km}^{cs}(x),$$

$$p_2=P_2(x_0,t)d(x_0,x)d(y_0,y)=P_2(x_0,t)\lim_{e\to 0}\frac{4}{l_1l_2}\sum_{k=0}^{\infty}\sum_{m=1}^{\infty}c_{km}(e)F_{km}^{sc}(x_0)F_{km}^{sc}(x), \tag{8}$$

where $F_{km}^{ss}(x)=\sin l_{1k}x\sin l_{2m}y$; $F_{km}^{cs}(x,y)=\cos l_{1k}x\sin l_{2m}y$; $F_{km}^{sc}(x)=\sin l_{1k}x\cos l_{2m}y$; $F_{km}^{cc}(x)=\cos l_{1k}x\cos l_{2m}y$; $l_{1k}=kp/l_1$; $l_{2m}=mp/l_2$; $c_{km}(e)=j(l_{1k}e)/2$, if $k\geq 1, m=0$, $c_{km}(e)=j(l_{2m}e)/2$, if $k=0, m\geq 1$, $c_{km}(e)=j(l_{1k}e)j(l_{2m}e)$, if $k\geq 1, m\geq 1$; $j(l_{1k}e)$, $j(l_{2m}e)$ – are summarizing sequences, namely $j(m)=(\sin m)^2\big/m^2$ [1].

Solution of the system equations (4), fulfilling conditions (5), (7), in accordance with (8), is searched in the form of generalized sums of trigonometric series

$$\bar{T}(x_0,x,t)=\int_0^t U_1^0(x_0,x,t-h)P_0(x_0,h)dh, \tag{9}$$

$$\bar{u}_1(x_0,x,t)=U_1^1(x_0,x)\frac{P_1(x_0,t)}{B}+U_1^2(x_0,x)\frac{P_2(x_0,t)}{B}+\int_0^t U_1^0(x_0,x,t-h)P_0(x_0,h)dh,$$

$$\bar{u}_2(x_0,x,t)=U_2^1(x_0,x)\frac{P_1(x_0,t)}{B}+U_2^2(x_0,x)\frac{P_2(x_0,t)}{B}+\int_0^t U_2^0(x_0,x,t-h)P_0(x_0,h)dh,$$

$$\frac{\bar{N}_{11}(x_0,x,t)}{B}=N_{11}^1(x_0,x)\frac{P_1(x_0,t)}{B}+N_{11}^2(x_0,x)\frac{P_2(x_0,t)}{B}+\int_0^t N_{11}^0(x_0,x,t-h)P_0(x_0,h)dh,$$

$$\frac{\bar{N}_{22}(x_0,x,t)}{B}=N_{22}^1(x_0,x)\frac{P_1(x_0,t)}{B}+N_{22}^2(x_0,x)\frac{P_2(x_0,t)}{B}+\int_0^t N_{22}^0(x_0,x,t-h)P_0(x_0,h)dh,$$

$$\frac{\bar{N}_{12}(x_0,x,t)}{B}=N_{12}^1(x_0,x)\frac{P_1(x_0,t)}{B}+N_{12}^2(x_0,x)\frac{P_2(x_0,t)}{B}+\int_0^t N_{12}^0(x_0,x,t-h)P_0(x_0,h)\,dh\ ,$$

where $D_{km}^2=l_{1k}^2+l_{2m}^2$; $G(x_0,x,t)=\frac{4}{l_1 l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}c_{km}(e)e^{-(a^2D_{km}^2+a)t}F_{km}^{ss}(x_0)F_{km}^{ss}(x)$;

$$U_1^1(x_0,x)=\frac{4}{l_1l_2}\lim_{e\to 0}\sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\frac{c_{km}(e)}{D_{km}^4}\left(l_{1k}^2+\frac{2}{1-n}l_{2m}^2\right)F_{km}^{cs}(x_0)F_{km}^{cs}(x)\,;$$

$$U_1^2(x_0,x)=-\frac{4}{l_1l_2}\frac{1+n}{1-n}\lim_{e\to 0}\sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\frac{c_{km}(e)\,l_{1k}\,l_{2m}}{D_{km}^4}F_{km}^{sc}(x_0)F_{km}^{cs}(x)\,;$$

$$U_2^1(x_0,x)=-\frac{4}{l_1l_2}\frac{1+n}{1-n}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=0}^{\infty}\frac{c_{km}(e)\,l_{1k}\,l_{2m}}{D_{km}^4}F_{km}^{cs}(x_0)F_{km}^{sc}(x)\,;$$

$$U_2^2(x_0,z)=\frac{4}{l_1l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=0}^{\infty}\frac{1}{D_{km}^4}\left(l_{2m}^2+\frac{2}{1-n}l_{1k}^2\right)F_{km}^{sc}(x_0)F_{km}^{sc}(x)\,;$$

$$N_{11}^1(x_0,x)=-\frac{4}{l_1l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=0}^{\infty}\frac{c_{km}(e)\,l_{1k}\left(l_{1k}^2+(2+n)\,l_{2m}^2\right)}{D_{km}^4}F_{km}^{cs}(x_0)F_{km}^{ss}(x)\,;$$

$$N_{11}^2(x_0,x)=\frac{4}{l_1l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\frac{c_{km}(e)\,l_{2m}\left(l_{1k}^2-n\,l_{2m}^2\right)}{D_{km}^4}F_{km}^{sc}(x_0)F_{km}^{ss}(x)\,;$$

$$N_{22}^1(x_0,x)=\frac{4}{l_1l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\frac{c_{km}(e)\,l_{1k}\left(l_{2m}^2-n\,l_{1k}^2\right)}{D_{km}^4}F_{km}^{cs}(x_0)F_{km}^{ss}(x)\,;$$

$$N_{22}^2(x_0,x)=-\frac{4}{l_1l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\frac{c_{km}(e)\,l_{2m}\left(l_{2m}^2+(2+n)\,l_{1k}^2\right)}{D_{km}^4}F_{km}^{sc}(x_0)F_{km}^{ss}(x)\,;$$

$$U_1^0(x_0,x,t)=-\frac{4(1+n)}{l_1l_2}\lim_{e\to 0}\sum_{k=0}^{\infty}\sum_{m=1}^{\infty}\frac{c_{km}(e)\,l_{1k}}{D_{km}^2}e^{-(a^2D_{km}^2+a)t}F_{km}^{ss}(x_0)F_{km}^{cs}(x)\,;$$

$$U_2^0(x_0,x,t)=-\frac{4(1+n)}{l_1l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=0}^{\infty}\frac{c_{km}(e)\,l_{2m}}{D_{km}^2}e^{-(a^2l_{km}^2+a)t}F_{km}^{ss}(x_0)F_{km}^{sc}(x)\,;$$

$$N_{11}^0(x_0,x,t)=-\frac{4\left(1-n^2\right)}{l_1l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\frac{c_{km}(e)\,l_{2m}^2}{D_{km}^2}e^{-(a^2D_{km}^2+a)t}F_{km}^{ss}(x_0)F_{km}^{ss}(x)\,;$$

$$N_{22}^0(x_0,x,t)=-\frac{4\left(1-n^2\right)}{l_1l_2}\lim_{e\to 0}\sum_{k=1}^{\infty}\sum_{m=1}^{\infty}\frac{c_{km}(e)\,l_{1k}^2}{D_{km}^2}e^{-(a^2D_{km}^2+a)t}F_{km}^{ss}(x_0)F_{km}^{ss}(x)\,;$$

$$N_{12}^1(x_0,x)=\frac{4}{l_1l_2}\lim_{e\to 0}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{c_{km}(e)\,l_{2m}\left(l_{2m}^2-n\,l_{1k}^2\right)}{D_{km}^4}F_{km}^{cs}(x_0)F_{km}^{cc}(x)\,;$$

$$N_{12}^2(x_0,x)=\frac{4}{l_1l_2}\lim_{e\to 0}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{c_{km}(e)\,l_{1k}\left(l_{1k}^2-n\,l_{2m}^2\right)}{D_{km}^4}F_{km}^{sc}(x_0)F_{km}^{cc}(x)\,;$$

$$N_{12}^0(x_0,x,t)=\frac{4\left(1-n^2\right)}{l_1l_2}\lim_{e\to 0}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\frac{c_{km}(e)\,l_{1k}\,l_{2m}}{D_{km}^2}e^{-(a^2D_{km}^2+a)t}F_{km}^{ss}(x_0)F_{km}^{cc}(x)\,.$$

Solving of the defined fixed task can be obtained from (9) by means of integral calculation under condition $P_0(x,t)=P_0(x)$ and transfer to the limit if $t\to\infty$.

Let us consider correlation (9) for the depiction of the problem solving in the integral form on heating and loading of the plate placed along line L «fictional» thermal sources and the intensity efforts $P_0(x,t),\ P_1(x,t),\ P_2(x,t)$, as well as definition of integral equations of the problem on the heating of triangular plate

$$T(x,t)=\int_0^t\int_L G(x_0,x,t-h)P_0(x_0,h)\,dl(x_0)\,dh\ ,$$

$$u_1(x,t)=\int_L\left[U_1^1(x_0,x)\frac{P_1(x_0,t)}{B}+U_1^2(x_0,x)\frac{P_2(x_0,t)}{B}\right]dl(x_0)+$$

$$+\int_0^t\int_L U_1^0(x_0,x,t-h)P_0(x_0,h)\,dl(x_0)\,dh\,,$$

$$u_2(x,t)=\int_L\left[U_2^1(x_0,x)\frac{P_1(x_0,t)}{B}+U_2^2(x_0,x)\frac{P_2(x_0,t)}{B}\right]dl(x_0)+$$

$$+\int_0^t\int_L U_2^0(x_0,x,t-h)P_0(x_0,h)\,dl(x_0)\,dh\,,$$

$$\frac{N_{11}(x,t)}{B}=\int_L\left[N_{11}^1(x_0,x)\frac{P_1(x_0,t)}{B}+N_{11}^2(x_0,x)\frac{P_2(x_0,t)}{B}\right]dl(x_0)+$$

$$+\int_0^t\int_L N_{11}^0(x_0,x,t-h)P_0(x_0,h)\,dl(x_0)\,dh\,,$$

$$\frac{N_{22}(x,t)}{B}=\int_L\left[N_{22}^1(x_0,x)\frac{P_1(x_0,t)}{B}+N_{22}^2(x_0,x)\frac{P_2(x_0,t)}{B}\right]dl(x_0)+$$

$$+\int_0^t\int_L N_{22}^0(x_0,x,t-h)P_0(x_0,h)\,dl(x_0)\,dh\,,$$

$$\frac{N_{12}(x,t)}{B}=\int_L\left[N_{12}^1(x_0,x)\frac{P_1(x_0,t)}{B}+N_{12}^2(x_0,x)\frac{P_2(x_0,t)}{B}\right]dl(x_0)+$$

$$+\int_0^t\int_L N_{12}^0(x_0,x,t-h)P_0(x_0,h)\,dl(x_0)\,dh\,.$$

If we set obtained integral solutions in the task (6), we'll come to the following system of integral equations

$$\lim_{r\to-0}\int_0^t dh\int_L G(x_0,x_r,t-h)P_0^*(x_0,h)\,dl(x_0)=T_0^*(x,t),$$

$$\lim_{r\to-0}\int_0^t dh\int_L U_1^0(x_0,x_r,t-h)P_0^*(x_0,h)\,dl(x_0)+\lim_{r\to-0}\int_L U_1^1(x_0,x_r)P_1^*(x_0,t)\,dl(x_0)+$$

$$+\lim_{r\to-0}\int_L U_1^2(x_0,x_r)P_2^*(x_0,t)\,dl(x_0)=0,$$

$$\lim_{r\to-0}\int_0^t dh\int_L U_2^0(x_0,x_r,t-h)P_0^*(x_0,h)\,dl(x_0)+\lim_{r\to-0}\int_L U_2^1(x_0,x_r)P_1^*(x_0,t)\,dl(x_0)+ \qquad (10)$$

$$+\lim_{r\to-0}\int_L U_2^2(x_0,x_r)P_2^*(x_0,t)\,dl(x_0)=0,\ \ x\in L\,,$$

where $P_0^*(x_0,t)=\dfrac{P_0(x_0,t)}{T_e}$; $P_1^*(x_0,t)=\dfrac{P_1(x_0,t)}{BT_e a_T}$; $P_2^*(x_0,t)=\dfrac{P_2(x_0,t)}{BT_e a_T}$; $T_0^*(x,t)=\dfrac{T_0(x,t)}{T_e}$; T_e – normalizing factor.

Numerical scheme of the integral equations system solution (10) is based on the idea of collocation method $[3]$:

a) It should be observed that unknown system functions (10) get fixed values for each time interval $\left](z-1)Dt;\ zDt\right[$, $z=1,2,\mathbf{L}$.

b) Let's cut the curve L in parts n of equal length $Dl_i=2pb/n$ and choose two sets of middle points of these closed intervals $E_0=\{x_{0i}(x_{0i},y_{0i}),\ i=\overline{1,n}\}$, $x_{0i}\in DL_i$, $E=\{x_j(x_j,y_j),\ j=\overline{1,n}\}$, $x_j\in DL_j$, where $x_{0i}=l_1/2-b_1\cos J_{0i}$, $y_{0i}=l_2/2-b_2\sin J_{0i}$, $x_j=l_1/2-b_1\cos J_j$, $y_j=l_2/2-b_2\sin J_j$ $J_{0i}=(2i-1)p/n$, $J_j=(2j-1)p/n$. These sets are a set of points of cuts and s set of refrerred points (points of "collocations"), respectively.

c) Each interval along line L in the equations (10) can be written in the form of sum n of integrals along parts DL_i. Since $r\neq 0$ and kernel functions of the given integrals are quite smooth functions, the mean value theorem can be applied. Thus, we state that unknown densities within each part of curve are constant values $P_0^*(x_0,t)=P_{0i}^{*z}$, $P_1^*(x_0,t)=P_{1i}^{*z}$, $P_2^*(x_0,t)=P_{2i}^{*z}$, $x_0\in DL_i$, $t\in\left](z-1)\Delta t;z\Delta t\right[$, and points where the mean value of subintegral functions is achieved are the middle points of curve parts $x_{0i}(x_{0i},y_{0i})$, $i=\overline{1,n}$. Let's calculate integrals after time coordinate taking into consideration unknown functions stability.

d) Generalized sums of double series, setting kernel functions of in integrals are changed by partial sums of length N and $e=e_0/N^g$, де $0<g<1$, $e_0=const$.

e) Equation (3) is satisfied in the points $t_s=sDt,\ s=1,2,\mathbf{K}$, and $x_{jr}(x_{jr},y_{jr})$ with $r\to-0$ (on the left "sides" parts of cutting), where as boundary elements of sequences of given functions are changed by their variants with $r=r_0/n^g$, where $0<g<1$, $r_0=const$, n – is quantity of parts of curve cutting L.

As s result of these transformations, linear algebraic equation system is obtained relative to $3n$ unknown

$$\sum_{s=1}^{z}\sum_{i=1}^{n}G_N^s(x_{0i},x_{jr})Dl_i\,P_{0i}^{*s}=T_0^*(x,zDt),$$

$$\sum_{i=1}^{n}U_{1N}^1(x_{0i},x_{jr})Dl_i\,P_{1i}^{*z}+\sum_{i=1}^{n}U_{1N}^2(x_{0i},x_{jr})Dl_i\,P_{2i}^{*z}=-\sum_{s=1}^{z}\sum_{i=1}^{n}U_{1N}^{*s}(x_{0i},x_{jr})Dl_i\,P_{0i}^{*s},$$

$$\sum_{i=1}^{n}U_{2N}^1(x_{0i},x_{jr})Dl_i\,P_{1i}^{*z}+\sum_{i=1}^{n}U_{2N}^2(x_{0i},x_{jr})Dl_i\,P_{2i}^{*z}=-\sum_{s=1}^{z}\sum_{i=1}^{n}U_{2N}^{*s}(x_{0i},x_{jr})Dl_i\,P_{0i}^{*s},\quad j=\overline{1,n},\ z=1,2,\mathbf{K}\qquad(11)$$

where $G_N^s(x_{0i},x_{jr})=\dfrac{4}{l_1l_2}\displaystyle\sum_{k=1}^{N}\sum_{m=1}^{N}\frac{c_{km}(e_N)}{a^2l_{km}^2+a}e^{-(a^2D_{km}^2+a)t}\Big|_{sDt}^{(s-1)Dt}F_{km}^{ss}(x_{0i})F_{km}^{ss}(x_{jr})$;

$$U_{1N}^1(x_{0i},x_{jr})=\frac{4}{l_1l_2}\sum_{k=0}^{N}\sum_{m=1}^{N}\frac{c_{km}(e_N)}{D_{km}^4}\left(l_{1k}^2+\frac{2}{1-n}l_{2m}^2\right)F_{km}^{cs}(x_{0i})F_{km}^{cs}(x_{jr});$$

$$U_{1N}^2(x_{0i},x_{jr})=-\frac{4}{l_1l_2}\frac{1+n}{1-n}\sum_{k=0}^{N}\sum_{m=1}^{N}\frac{c_{km}(e_N)\,l_{1k}\,l_{2m}}{D_{km}^4}F_{km}^{sc}(x_{0i})F_{km}^{cs}(x_{jr});$$

$$U_{1N}^s(x_{0i},x_{jr})=-\frac{4(1-n)}{l_1l_2}\sum_{k=0}^{N}\sum_{m=1}^{N}\frac{c_{km}(e_N)\,l_{1k}}{D_{km}^2(a^2l_{km}^2+a)}e^{-(a^2D_{km}^2+a)t}\Big|_{sDt}^{(s-1)Dt}F_{km}^{ss}(x_{0i})F_{km}^{cs}(x_{jr});$$

$$U_{2N}^{1}\left(x_{0i},x_{jr}\right)=-\frac{4}{l_1 l_2}\frac{1+n}{1-n}\sum_{k=1}^{\infty}\sum_{m=0}^{\infty}\frac{c_{km}\left(e_N\right)l_{1k}\,l_{2m}}{D_{km}^{4}}F_{km}^{cs}\left(x_{0i}\right)F_{km}^{sc}\left(x_{jr}\right);$$

$$U_{2N}^{2}\left(x_{0i},x_{jr}\right)=\frac{4}{l_1 l_2}\sum_{k=1}^{N}\sum_{m=0}^{N}\frac{1}{D_{km}^{4}}\left(l_{2m}^{2}+\frac{2}{1-n}\,l_{1k}^{2}\right)F_{km}^{sc}\left(x_{0i}\right)F_{km}^{sc}\left(x_{jr}\right);$$

$$U_{2N}^{s}\left(x_{0},x_{jr}\right)=-\frac{4(1-n)}{l_1 l_2}\sum_{k=1}^{N}\sum_{m=0}^{N}\frac{c_{km}\left(e_N\right)l_{2m}}{D_{km}^{2}(a^2 l_{km}^{2}+a)}\,e^{-(a^2 D_{km}^{2}+a)t}\Big|_{sDt}^{(s-1)Dt}F_{km}^{ss}\left(x_{0i}\right)F_{km}^{sc}\left(x_{jr}\right);$$

$x_{0i}\left(x_{0i},y_{0i}\right)$, $x_{jr}\left(x_{jr},y_{jr}\right)$ – points where the coordinates might be calculated according to formulas (1) and (3).

For the functions determining thermoelastic state of plate at free points, we will get from (9) the analogical (11) discreet quantifiers.

Effective numerical equation system solutions (11) for the case $l_1=p$, $l_2=2p$, $b_1=3p/8$ are designed with the following parameters values: $r<0{,}005$; $e<0{,}001$; $n=80$; $N=800$. Considering low values $\max\left\{b_i/l_j\right\}<0{,}1$ the given model is exact model of thermoelastic state of unlimited plate with inclusions.

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PROJECT MANAGEMENT OF CARBON DIOXIDE PRODUCED DURING OPERATION OF LOW POWER BOILER IN SMALL TOWN IN CENTRAL EUROPE

The main point of the article is reducing carbon dioxide. The ways of carbon sequestration for energy and mining corporations are presented. It is shown that joint action to protect the environment can bring tangible benefits.

Key words: Sequestration, carbon dioxide emission.

Розглянуто зменшення обсягів викиду двоокису вуглецю. Представлено способи секвестрації вуглецю для енергетичних і гірничодобувних корпорацій. Показано, що спільні дії щодо захисту навколишнього середовища можуть принести відчутну користь.

Ключові слова: секвестрація, викид двоокису вуглецю.

Introduction

In recent times global warming topics are getting very trendy on the media. They usually say that the main reason for global warming is the excessive carbon dioxide emissions caused by the rapid development of civilization. For the existing state of things they blame large industrial plants, energy facilities or processing plants that are using coal or other fossil fuels for combustion and then pollute the environment with carbon dioxide. On the other hand, global warming skeptics argue that putting