COMPUTATIONAL PROBLEMS OF ELECTRICAL ENGINEERING

Vol. 2, No. 2, 2012

CALCULATION OF ELECTROMAGNETIC PROCESSES FOR A TURBOGENERATOR WITH EQUIVALENT STATOR AND ROTOR TOOTH ZONES IN NO-LOAD REGIME

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Abstract: On the basis of Maxwell's equations with respect to potentials, calculations of electromagnetic processes in movable and immovable areas of a turbogenerator cross-section in transformed and physical coordinate systems for no-load regime have been done. It has been shown that employing the method of equivalentization of the device tooth zones by continuous anisotropic media and ignoring the coordinate systems in which the process is considered in terms of mathematical field models lead to misrepresentation of real electromagnetic phenomena.

Key words: electromagnetic field, Maxwell's equation, movable media, coordinate systems, tooth zones.

1. Introduction

The problem of developing mathematical field models for electrodynamic devices is of great importance for the theory and practice of designing, developing, manufacturing and maintaining electrical objects of the kind. Here also belongs a turbogenerator being the main source of electric energy in the world.

The solution to the task set before us is impeded by some theoretical and practical problems caused by a need to calculate an electromagnetic field in movable and immovable, linear and nonlinear, isotropic and anisotropic media [1]. In modern scientific literature this problem is paid much attention to for the purpose of developing some approaches to determination of electromagnetic processes in electrodynamic devices by exclusively using the theory of electromagnetic field.

The most practically used method of the development of mathematical field models implies application of equivalent substitution circuits with lumped parameters to the description of an electrodynamic object structure. The parameters of the circuits can be calculated from spatial distributions of an electromagnetic field obtained for reciprocal fixed positions of moving and stationary elements of a device under predetermined (given) magnetomotive forces [2, 3]. The models of this kind do not imply taking a vortical component of the field into account, a possibility of analyzing dynamic electromagnetic processes as well as electromagnetic phenomena occurring in a time slot of the device zones on the basis of Maxwell's equations. This causes a significant misrepresentation of real electromagnetic processes in electrodynamic objects**.**

Many scientific works dedicated to calculations of an electromagnetic field in electrodynamic structures ignore the coordinate systems in which basic calculation values are formed in basic vectors or potentials [4, 5]. This results from the use of timeless techniques to determine unknown variables. Such an approach makes it impossible to analyze electromagnetic phenomena both in movable and immovable elements of a device in a time domain in appropriate coordinate systems. But the electromagnetic process in movable media is known to always depend on a reference frame in which it is considered.

Given the major disadvantages of the above analyzed methods, let us consider some ways of developing mathematical field models for a turbogenerator with equivalent stator and rotor zones in transformed and physical coordinate systems in the case of a no-load regime. They are based on Maxwell's equations with respect to potentials, and involve both direct time integration of the field equations and reference systems the process is described in.

2. Statement of the problem

Since the development of a 3-D dynamical model of a turbogenerator on the basis of the theory of electromagnetic field is a complicated task, to ease the problem let us consider a 2-D model with field flat-parallelism along the device axis accepted. The models of the class do not take electromagnetic phenomena in the objects' side face zones into consideration, but as the given model is designed to reproduce a no-load regime of a turbogenerator such an assumption is acceptable. Given the complexity of the object design, and in order to simplify its mathematical description, we are to replace the stator and rotor tooth zones in the model by the equivalent nonlinear anisotropic media with the electromagnetic characteristics above. Their properties can be determined by making use of the known methods offered by the theory of electromagnetic circuits to calculate parameters of substitution circuits of parallel and series connection of electrical and magnetic resistances. Such an approach being more progressive for analyzing electromagnetic processes than the use of equivalent substitution schemes is widely used to develop mathematical field models of sophisticated electrical devices [6].

Experience of the development of mathematical field models for electromechanical devices, calculation algorithms as well as analysis of the computer simulation results show that it is practically impossible to create a uniform mathematical field model of a turbogenerator intended for calculating all possible operating modes of a real device. Investigations of operating conditions are to be carried out employing several models: a model intended for calculating a no-load regime, a model for computer simulation of a short circuit regime, and a model for the reproduction of operating modes of a turbogenerator. This is due not only to the peculiarity of calculation of an electromagnetic field in movable media [1] and some restrictions on use of the field approach to the description of electromagnetic phenomena only in a single link of a complicated electric circuit (of a turbogenerator), but specifically to the necessity of following the laws of switching both in methods of the theory of field and those of the theory of circuits.

As, in the field consideration, the results of no-load regime calculations are initial data for computer simulation of other modes of the device, let us develop mathematical field models intended for simulating a no-load regime. Besides, there exist some peculiarities of electromagnetic processes in such, at first sight, a simple no-load regime of a turbogenerator.

Let us consider the development of a 2-D mathematical field model of a no-load regime turbogenerator in transformed and phase coordinate systems.

Fig. 1 depicts the location of the available media of the turbogenerator cross-section with equivalent stator and rotor tooth zones, where 1 stands for the massive rotor body; 2 represents the equivalent rotor tooth zone; 3 is the air gap between the stator and rotor; 4 is the equivalent stator tooth zone; 5 stands for the stator body; 6 is the air gap outside the turbogenerator. If we consider a zone scheme given in Fig.1 concerning radii of the device design elements, then R_1 is the external edge radius of the rotor slot; R_2 is the external radius of the rotor; R_3 is the internal radius of the stator; R4 is the external edge radius of the stator slot; R_5 is the external radius of the stator; and R_6 is the nominal radius of the air gap over the turbogenerator in which the electromagnetic process is calculated.

The main geometric dimensions of cross-sectional areas of the real turbogenerator TGV-500 corresponding to Fig. 1 are given in Fig. 2.

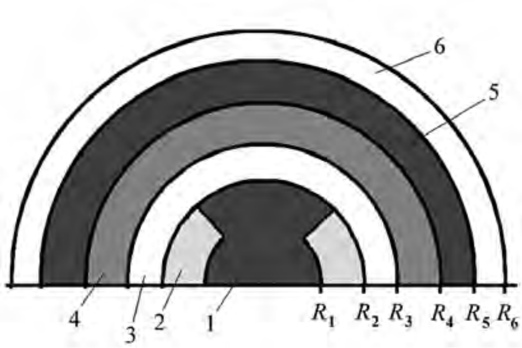

Fig. 1. Zones of turbogenerator cross section in which field calculations are carried out.

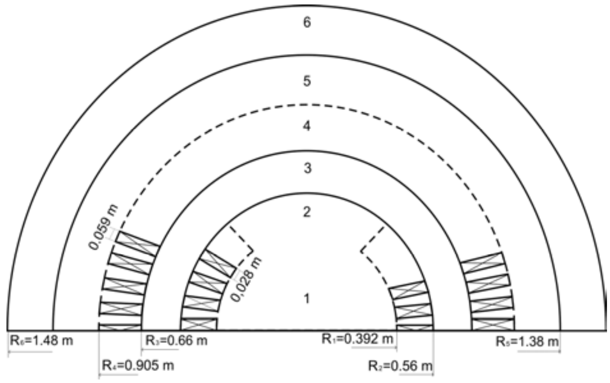

Fig. 2. Main geometrical dimensions of cross-sectional areas of the turbogenerator TGV-500.

3. A turbogenerator mathematical model in the transformed system of coordinates

Calculations of electromagnetic processes in electrical devices can be done by Maxwell's equations either in the base vectors **H**, **E**, **B**, **D,** or in the potentials **A**,φ. Choosing the variables depends on the peculiarities of the device design, its symmetry relative to the windings as well as the necessity to minimize the number of equations and the possibility to create both boundary and edge conditions for them.

Taking the periodicity of electromagnetic processes on the pole division of a turbogenerator into account, it is acceptable to calculate electromagnetic phenomena only in the sector of a device cross-section that corresponds to the pole division of the object. To simplify the task of determining boundary conditions and decreasing the number of main equations, let us apply the equation of electromagnetic field written with respect to potentials. To introduce a single-valued relation between functions of the vector A and scalar potentialφ, let us choose the known calibration $\varphi = 0$[6].

The only mathematically possible and physically reasonable way of the transfer of electromagnetic values from one reference frame to another is the Lorentz-Poincare transformation which is written with respect to the vector of electric field intensity.

$$\mathbf{E}' = \mathbf{E} + \boldsymbol{\upsilon} \times \mathbf{B}, \qquad (1)$$

where $\mathbf{E}'$ is the vector of electric field intensity in the moving system of coordinates; $\mathbf{E}$ is the vector of electric field intensity in the stationary system of coordinates; $\boldsymbol{\upsilon}$ is the vector of linear velocity of the moving coordinates system relative to the stationary one; $\mathbf{B}$ is the vector of magnetic induction in the immovable reference frame.

As we see from the equation (1) the difference between the values of electric field intensity vectors in the moving and fixed systems of coordinates reaches $(\boldsymbol{\upsilon} \times \mathbf{B})$ which defines the character of electromagnetic phenomenon occurring in a moving environment relative to an observer being in an immovable frame of reference. That is why the key moment when calculating electromagnetic processes in moving elements of electrodynamics devices by means of the theory of electromagnetic field is choosing a system of coordinates in which the main calculated values are written.

If we base on the expression (1), we shall obtain the following ratio for converting the values of vector potential function from one reference frame to another [1]

$$\frac{\partial \mathbf{A}'}{\partial t} = -\Gamma^{-1}\left(\nabla \times \left(\mathrm{N}\nabla \times \mathbf{A}\right) + \boldsymbol{\upsilon} \times \nabla \times \mathbf{A}'\right). \qquad (2)$$

The ratio (2) cannot be used for direct calculation of electromagnetic processes as it contains two unknown variables $\mathbf{A}, \mathbf{A}'$ - the functions of vector potential in moving and fixed systems of coordinates. It can be applied only to convert the value of $\mathbf{A}$ (according to (1)) from once frame of reference to another [1]. It is possible for the electromagnetic quantities to be calculated only in such a coordinate system with respect to which main equations can definitely be formed. The equation (2) is not appropriate to be applied in this particular case [1]. From the practical point of view, the calculation of electromagnetic field can be carried out either in moving or fixed coordinate systems, or simultaneously in moving and fixed frames of reference, with the boundary conditions being recalculated.

As there is a necessity to do calculation of the electromagnetic field in movable media, the given model implies reducing an immovable coordinate system of a stator to movable coordinates of a rotor. In this way we can dispose of the actual physical movement of the media, but at that the obtained electromagnetic process in the stator tooth zone as well as in its body corresponds to the transformed reference frame rather than to the physical one

In the model considered, the tooth zone of the stator has been replaced by a continuous anisotropic medium, and it is isotropic along the angular coordinate, that is why the use of the transformed system of coordinates is advisable, as this enables the calculations to be considerably simplified without ignoring the peculiarities of electromagnetic phenomena both in the tooth zone and the body of the stator in the physical system of coordinates. Transition to the physical reference frame occurs by the certain angular coordinate displacement of the obtained distributions of the electromagnetic quantities that corresponds to the real reciprocal position of the stator and rotor in the given fixed moment of time *t* [1]. Hence, the calculations of electromagnetic processes in the body and equivalent medium of the rotor tooth zone as well as in the air gap between the rotor and stator will be carried out in the physical frame of reference related to the rotor while in the equivalent tooth zone of the stator, in its body, and in the air gap beyond the turbogenerator the same procedure will be performed in the transformed system of coordinates reduced to the coordinates of the movable rotor.

The equation for calculating an electromagnetic field in movable equivalent media (with given external currents) of the rotor tooth zones is represented in the following form [6].

$$\frac{\partial \mathbf{A}}{\partial t} = -\Gamma^{-1}\left(\nabla \times \left(\mathrm{N}\nabla \times \mathbf{A}\right) \pm \delta\right), \qquad (3)$$

where $\mathbf{A}$ is the vector potential of electromagnetic field in the coordinate system of a moving rotor; $\boldsymbol{\Gamma}$ is the matrix of static electrical conductivities; N is the matrix of static inverse magnetic penetrability of the medium; δ is the extraneous current density vector.

To calculate electromagnetic processes in the rotor body we can use the following expression [6]

$$\frac{\partial \mathbf{A}}{\partial t} = -\Gamma^{-1}\nabla \times \left(\mathrm{N}\nabla \times \mathbf{A}\right), \qquad (4)$$

where $\mathbf{A}$ is the function of electromagnetic field vector potential in the coordinate system of a moving rotor.

The electromagnetic process in the air gap zone between the stator and rotor can be calculated by means of the following ratio

$$0 = \nu_0 \nabla \times \nabla \times \mathbf{A}, \qquad (5)$$

where ν_0 is the inverse magnetic air penetrability.

The calculation of an electromagnetic field both in a tooth zone and body of the stator in the transformed coordinate system for a no-load regime is carried out by the expression below [1]

$$0 = \nabla \times \mathrm{N}\nabla \times \mathbf{A}', \qquad (6)$$

where $\mathbf{A}'$ is the function of vector potential of the electromagnetic field in the transformed coordinate system of the stator reduced to the moving rotor.

The electromagnetic process in the air gap zone outside the turbogenerator can be calculated by the following equation

$$0 = \nu_0 \nabla \times \nabla \times \mathbf{A}'. \qquad (7)$$

The nonlinear electromagnetic characteristics of the stator and rotor materials in the mathematical field model of a turbogenerator are taken into consideration by means of the following splines

$$\nu(B)=\sum_{m=1}^{3} a_i^{(k)}(B_k-B)^m\ ,\ \ k=1,2,\ldots,n, \qquad (8)$$

where n is the number of segmentations along the B axis, with the module value of a magnetic induction vector in the spatial grid nodes of the cylindrical coordinate system being obtained from the following ratios

$$B_r=\frac{1}{r}\frac{\partial A}{\partial\alpha};\quad B_\alpha=-\frac{\partial A}{\partial r};\quad B=\sqrt{B_r^2+B_\alpha^2}\ , \qquad (9)$$

where B_r, B_α, B are the radial tangential components and module of a magnetic induction vector in the grid nodes of the physical and transformed coordinate system.

The recalculation of anisotropic electromagnetic characteristics of the equivalent stator and rotor tooth zones as well as the zone of laminated stator body is carried out by means of the known ratios for the determination of series and parallel connection of electric and magnetic resistances [6]

$$\nu_r=\frac{d_f+\nu_0\cdot d_0/\nu}{d_f+d_0}\cdot\nu;\quad \nu_r=\frac{d_f+d_0}{d_f+\nu\cdot d_0/\nu_0}\cdot\nu;$$

$$\gamma=\frac{\gamma_f\, d_f+\gamma_{Cu}\cdot d_0}{d_f+d_0}, \qquad (10)$$

where γ_f is the electrical conductivity of the rotor material; γ_{Cu} is the copper conductivity; d_f, d_0 are either the width of the tooth and slot, or the ferromagnetic sheet and isolation of the laminated stator.

Since the main electromagnetic field equations written with reference to the vector potential contain the second spatial derivative with respect to **A**, and the calculation of electromagnetic quantities is performed only on the pole turbogenerator division, it is reasonable that the boundary condition given below be used so that the boundary conditions on a level of the second spatial derivative of the vector potential can more fully be taken into consideration.

$$\left.\frac{\partial H_r}{\partial\alpha}\right|_{\alpha=0}=-\left.\frac{\partial H_r}{\partial\alpha}\right|_{\alpha=180}. \qquad (11)$$

This ratio is theoretically grounded and physically valid as the electromagnetic process on the pole turbogenerator division is periodical not only on the level of the first spatial derivative of the electromagnetic field vector potential, but also on the level of the higher derivatives of this function.

The boundary conditions along the radii of the rotor body zone, equivalent rotor tooth zone and air gap between the stator and rotor on the turbogenerator pole division in the physical coordinate system of the rotor, with the equations (9) and (11) being considered, can be represented as [7]

$$A_{k=1}=2A_{k=2}+2A_{k=n-1}-A_{k=3}-A_{k=n-2}-A_{k=n};$$

$$A_{k=n+1}=A_{k=2}+A_{k=4}+A_{k=n-1}-2A_{k=3}-2A_{k=n},\ (12)$$

whereas along the radii of the equivalent stator tooth zone, the stator body and the air gap outside the turbogenerator correspondingly as [7]

$$A'_{k=1}=2A'_{k=2}+2A'_{k=n-1}-A'_{k=3}-A'_{k=n-2}-A'_{k=n};$$

$$A'_{k=n+1}=A'_{k=2}+A'_{k=4}+A'_{k=n-1}-2A'_{k=3}-2A'_{k=n},\ (13)$$

where k is the index corresponding to certain nodes of the spatial grid along the angular coordinate.

The boundary conditions for the equations of vector potential outside the turbogenerator are determined from the ratio below

$$A'_{i=m+1}=2A'_{i=m}-A'_{i=m-1}, \qquad (14)$$

where i is the index corresponding to the nodes of the spatial grid in the cylindrical system of coordinates along the radius.

On the internal boundaries of the tooth zone of the rotor and its body, the body of the rotor and the air gap between the stator and rotor, the tooth zone of the rotor and the air gap between the stator and rotor, the boundary conditions are found from the expression

$$A_i=\frac{\nu_{i-1}A_{i-1}+\nu_{i+1}A_{i+1}}{\nu_{i-1}+\nu_{i+1}}, \qquad (15)$$

and between the tooth zone of the stator and its body, the body of the stator and the air zone by means of

$$A'_i=\frac{\nu_{i-1}A'_{i-1}+\nu_{i+1}A'_{i+1}}{\nu_{i-1}+\nu_{i+1}}, \qquad (16)$$

in which the indexes i point to the numbers of the nodes that correspond to the media distribution boundary.

The value of the rotor's winding current is determined by the equation

$$\frac{di_f}{dt}=\left(u_f-r_f i_f-\frac{d\psi_f}{dt}\right)\Big/L_f\ , \qquad (17)$$

where

$$\frac{d\psi_f}{dt}=w_f k_f l_r\sum_{i=1}^{n}\frac{\partial A_{Ri}}{\partial t}, \qquad (18)$$

with w_f being the number of the rotor's windings; l_r being the axis winding length; A_{Ri} being the value of the function of vector potential in the coordinate system of the rotor in the grid nodes being located in the winding**;** k_f being the coefficient involving the number of the nodes along the angular coordinate α which get into the rotor's winding zone.

The determination of the voltage in the stator winding phases is performed on the basis of the ratios given below

$$u_i=\frac{d\Psi_i}{dt}=w_i k_i l\sum_{m=1}^{n}\frac{\partial A'_{Si}}{\partial t},\quad i=\mathrm{A,\,B,\,C}, \qquad (19)$$

where u_i is the voltage in the stator's windings; w_i is the number of stator armature windings in each phase; l is the axial winding length; A'_{Si} is the value of the vector potential function in the nodes of a dimensional discretization grid connected to the transferred stator coordinate system within the windings zone.

4. Mathematical model of a turbogenerator in physical system of coordinates

Let us consider the development of a mathematical field model of a turbogenerator in a physical system of coordinates. The specific feature of the model is that electromagnetic processes in a movable rotor are considered in the coordinate system of a moving rotor, while in a stator this is done in the coordinate system of an immovable stator. The given model implies the realization of physical movement of the rotor relative to the stator along the angular coordinate.

The calculation of electromagnetic phenomena in the equivalent tooth zone and the massive rotor body is performed by the expressions (1) and (2) respectively. In these equations the function of vector potential in the electromagnetic field **A** is connected with the rotor coordinates system. In all the other zones of the turbogenerator cross-section, A-values belong to the coordinate system of an immovable stator.

The electromagnetic processes in the tooth zone and in the stator body can be found from the ratio [1]

$$0 = \nabla \times \mathrm{N} \nabla \times \mathbf{A}, \qquad (20)$$

whereas the electromagnetic phenomena in the air gaps zones between the stator and rotor, and outside the turbogenerator can be calculated by the expression

$$0 = \nu_0 \nabla \times \nabla \times \mathbf{A}, \qquad (21)$$

where **A** is the function of vector potential of the electromagnetic field in the coordinate system of an immovable stator.

The boundary condition along the radii of the turbogenerator cross-section zones on the pole division of the turbogenerator in physical coordinate systems takes the following form [7]

$$A_{k=1} = 2A_{k=2} + 2A_{k=n-1} - A_{k=3} - A_{k=n-2} - A_{k=n};$$
$$A_{k=n+1} = A_{k=2} + A_{k=4} + A_{k=n-1} - 2A_{k=3} - 2A_{k=n}, \qquad (22)$$

And outside the turbogenerator **it** (this)can be expressed as

$$A_{i=m+1} = 2A_{i=m} - A_{i=m-1}. \qquad (23)$$

On all the internal boundaries of the turbogenerator's cross-section zones, the boundary conditions are to be found on the basis of the equations (15).

The voltage values of the stator's winding phase are found from the expression

$$u_i = \frac{d\Psi_i}{dt} = w_i k_i l \sum_{m=1}^{n} \frac{\partial A_{Si}}{\partial t}, \quad i = \mathrm{A, B, C}, \qquad (24)$$

where A_{Si} is the vector potential of the electromagnetic field in the stator coordinate system.

All the other auxiliary mathematical ratios (6)–(8), (14), (15) are of the same form as in the previous model.

The considered mathematical field model of a turbogenerator in the phase coordinates implies reciprocal displacement of movable media in physical reference frames relative to the nodes of spatial discretization grids provided the ratio (22) is followed

$$\Delta\alpha = \omega \Delta t, \qquad (25)$$

where $\Delta\alpha$ is the angular step of the discretization grid; Δt is the step of time integration of the differential equations system; ω is the angular rotary speed.

5. Computation results

On the basis of the developed models there was carried out computer simulation of the transition process of a no-load regime of the real turbogenerator TGV-500 when the voltage of its excitation winding was equal to $u_f = 141$ V.

Fig. 3 represents the time dependence of excitation winding current i_f of the turbogenertor's rotor in the transition process of the no-load regime.

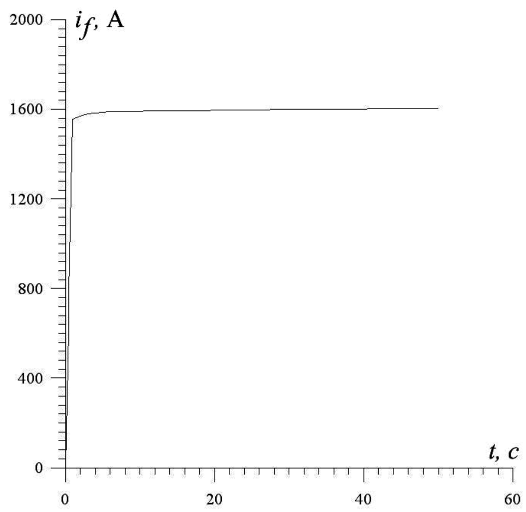

Fig. 3. Time values of an excitation winding current of the turbogenertor rotor.

Fig. 4–9 demonstrate the calculated spatial divisions of the magnetic induction vector module in the turbogenerator cross-section zones in the transformed and physical coordinate systems.

On the given spatial radius distribution, the coordinate grid nodes correspond to the following zones of the turbogenerator cross-section: 0÷54 is the ferromagnetic rotor body zone; 54÷75 is the equivalent rotor tooth zone; 75÷87 is the zone of an air gap between the stator and rotor; 87÷117 is the equivalent stator tooth zone; 117÷174 is the stator body zone; 174÷186 is the air gap outside the turbogenerator. The number of the discretization grid nodes along an angular coordinate on the pole division of the turbogenerator is equal to 192.

Fig. 4 depicts the spatial distribution of magnetic induction vector module in **a** coordinate system on the pole division of a turbogenerator when t = 1 sec of the transition process in a no-load regime obtained on the basis of the mathematical field model in the transformed system of coordinates.

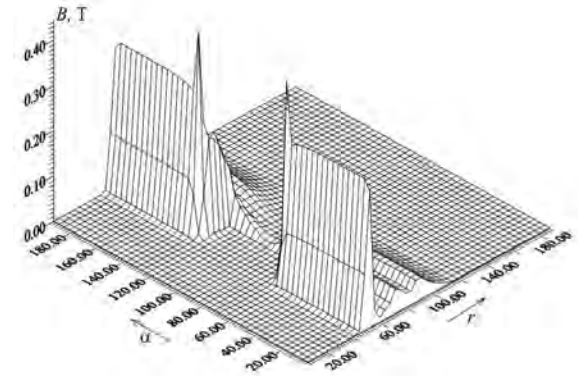

Fig. 4. Spatial distribution of magnetic induction vector module in transformed coordinate systems on the pole divisi on of a turbogenerator when t = 1 sec of transition process in a no-load regime.

Fig. 5 shows the spatial distribution of magnetic induction vector module in transformed coordinate systems on the turbogenerator pole division when t = 100 sec of transition process in a no-load regime obtained on the basis of the mathematical field model in the transformed system of coordinates.

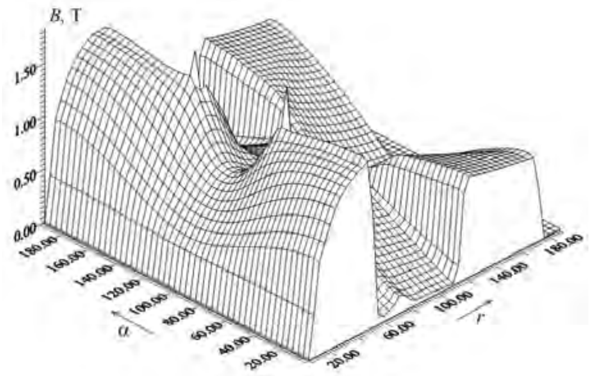

Fig. 5. Spatial distribution of magnetic induction vector module in transformed coordinate systems on turbogenerator pole division when t = 100 sec of transition process in a no-load regime.

Fig. 6 illustrates the spatial distribution of the module of a magnetic induction vector in the rotor coordinate system on the pole division of the turbogenerator when t = 650 sec of the transition process in a no-load regime, obtained on the basis of the field mathematical model in the transformed coordinate system.

Fig. 7 represents the spatial distribution of magnetic induction vector module on the pole division of the turbogenerator in the phase coordinate systems when t = 650,01543 sec of the transition process in a no-load regime obtained on the basis of the mathematical field model in the phase coordinates.

Fig. 8 shows the spatial distribution of the magnetic induction vector module on the pole division of the turbogenerator in the phase coordinate systems at the time when t = 650, 01766 sec of the transition process in a no-load regime obtained on the basis of the mathematical field model in the phase coordinates.

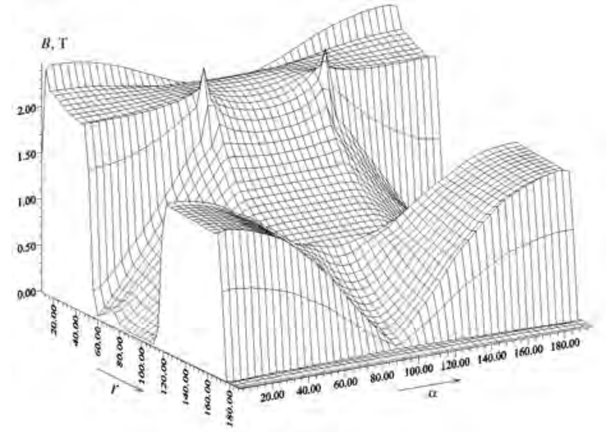

Fig. 6. Spatial distribution of the magnetic induction vector module in transformed coordinate systems on turbogenerator pole division when t = 650 sec of transition process in a no-load regime.

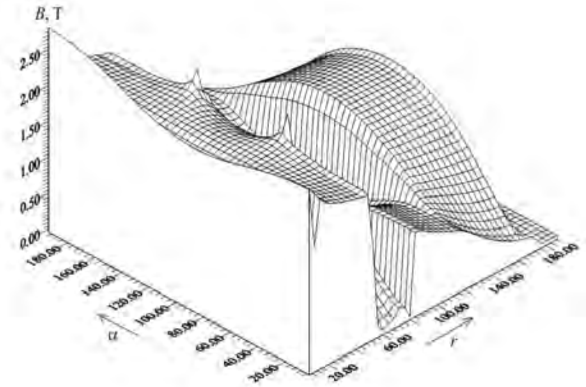

Fig. 7. Spatial distribution of the magnetic induction vector module on turbogenerator pole division in phase coordinate systems when t = 650,01543 sec of transition process in a no-load regime.

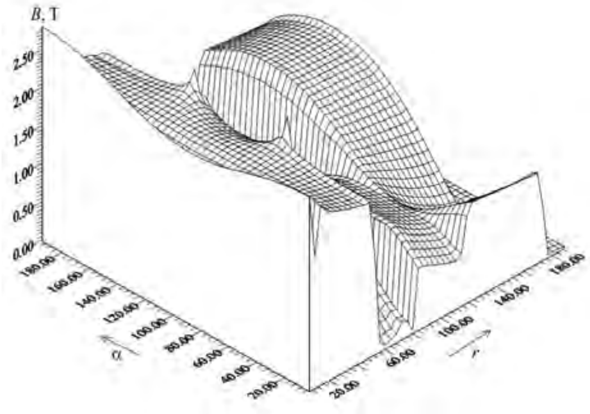

Fig. 8. Spatial distribution of magnetic induction vector module on turbogenerator pole division in phase coordinate systems when t = 650, 01766 sec of transition process in a no-load regime.

Fig. 9 demonstrates the spatial distribution of the magnetic induction vector module on the pole division of the turbogenerator with an conductive tooth zone of the stator in the phase coordinate systems at the time when t = 650, 01988 sec of the transition process in a no-load regime obtained on the basis of the mathematical field model in the phase coordinates.

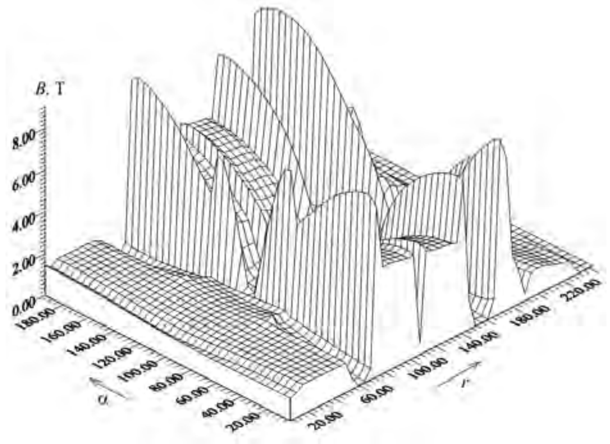

Fig. 9. Spatial distribution of magnetic induction vector module on turbogenerator pole division with an conductive stator tooth zoneз in phase coordinate systems when t = 650, 01988 sec of transition process in a no-load regime.

The obtained character of changing the turbogenerator's rotor excitation winding current i_f (Fig. 3) in the transition process of a no-load regime shows that when t = 1 sec the transition process in the winding is practically completed. While in Figure 4 we see that the transition electromagnetic process in the turbogenerator in a no-load regime when t = 1 sec has just started, and is still far to its end. Even if t = 100 c (Fig. 5) it is still going on. This confirms the importance of considering electromagnetic phenomena in different elements of the device construction (not only in the windings) when modeling even such a simple operating mode of the turbogenerator i.e. a no-load regime. The behaviour of electrodynamics devices in different modes is defined by electromagnetic processes in all their elements and not in windings only.

If we compare the results given in Fig. 6 and 7, then we can see that the spatial distribution of the magnetic induction vector module in the equivalent stator tooth zone and in the stator body (Fig. 6) is displaced relative to the angular coordinate for a value that corresponds to the real reciprocal location of the rotor and stator at the given fixed time (Fig. 7–8).

The data given in Fig. 6 are obtained by using the mathematical field model of a turbogenerator in the transformed coordinate system. It is easier to develop the model of this kind than that in the phase coordinates because of the absence of any physical medium movement in it. That is why, it is reasonable that the mathematical field model in a transformed coordinate system be used to calculate a no-load regime of a turbogenerator. Transition to a physical system of coordinates is realized by displacing electromagnetic quantities obtained for a certain angular coordinate [1]. This is verified by the results given in Fig. 7–8 obtained on the basis of the mathematical field model of a turbogenerator in the physical coordinate system. The relative erro for the difference of the results of spatial distributions of electromagnetic quantities on the pole division of the turbogenerator cross-section zones obtained on the basis of the two considered models for a random fixed time point of the stator and rotor reciprocal placement can achieve less than 1 %.

Special attention should be paid to the spatial distribution of the module of the magnetic induction vector in the turbogenerator cross-section zones (Fig. 9) calculated by means of the mathematical field model in the phase coordinates. Fig. 9 shows a large increase of the magnetic induction vector module in the equivalent stator tooth zones as well as a considerable magnetic skin-effect on both the surface of the massive rotor and the surface of its equivalent tooth zone. This is because the electromagnetic process in an equivalent stator tooth zone in the mathematical model of a turbogenerator in a physical coordinate system was determined by the expression given below

$$\frac{\partial \mathbf{A}}{\partial t} = -\Gamma^{-1}\nabla \times \left(\mathrm{N}\nabla \times \mathbf{A}\right), \tag{26}$$

as it is the very ratio to be used for describing electromagnetic phenomena in conductive zones of electrical devices [6].

In the turbogenerator no-load regime, the stator winding is open, i.e. there are no conductive currents in it. Since the mathematical model in phase coordinates involves physical displacement of a rotor (a source of turbogenerator magnetic field in the given mode) relative to a stator, as it occurs in a real object, this leads to changing values of the function of vector potential of the electromagnetic field in the stator tooth zone, i.e.

$$\frac{\partial \mathbf{A}}{\partial t} \neq 0\,. \tag{27}$$

This phenomenon, if the medium is conductive ($\Gamma \neq 0$), results in inducing an eddy component of the field (an eddy current). Thus, by using the expression (26) we can present a regime of symmetric short circuit of the stator winding rather than the no-load one of the turbogenerator. These are the results that demonstrate the spatial distribution shown in Fig. 9. The results we obtained just confirm the adequacy of the developed model, as in any conductive environment moving in a magnetic field, eddy currents will be induced.

To reproduce a no-load regime of a turbogenerator, the electromagnetic process in an equivalent tooth zone of a stator should be calculated by the expression (20) rather than (26), as the equation (20) implies the absence of conductive currents in the tooth zone of the stator functioning in the no-load regime. It was the very way of obtaining results presented in Fig. 7–8.

The availability of $\Gamma \neq 0$ in the equation (26) of the mathematical field model of a turbogenarator in a no-load regime in phase coordinate systems while describing electromagnetic processes in the tooth zone of the stator leads to reproduction of the winding short-circuit mode.

Fig. 10 shows time dependences of A-phase voltage of stator winding after completing the transition process for the linear (1) and nonlinear (2) cases of electromagnetic media characteristics achieved on the basis of the mathematical models in both the transferred and physical coordinate systems.

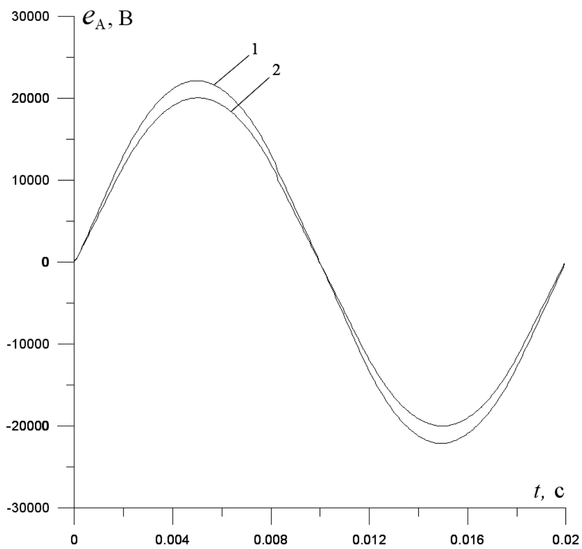

Fig. 10. Time values of A-phase voltage in the stator winding after completing the transition process for linear (1) and nonlinear (2) characteristics of electromagnetic media obtained on the basis of the models developed.

Some attention should also be paid to voltage curves shown in Fig. 10. According to specifications of the turbogenerator TGV-500, the phase voltage peak value equals 16300 V. The voltage peak value in the linear model is equal to 22000 V, while in the nonlinear one is 20000 V (Fig.10). Thus, the most accurate field model of a turbogenerator is the cause of a 35% error in a linear variant and a 23% error in a nonlinear one. No assumptions introduced by the theory of circuits can explain this. For a long time the results obtained could not be explained logically. Seeking the causes of such an error in the theory and in the calculation algorithm was a failure. Only after developing more detailed models accounting real tooth structures of a stator and rotor the identified problem was solved. Therefore, it must be emphasized that only the field mathematical model of a turbogenerator more fully taking into account a real structure of a device ensues calculations of high accuracy.

Using the method of replacement of complex structures of electrical devices by equivalent media actually changes the structure of the device, and in this case it is a mistake to expect real electromagnetic processes in changed structures of the objects to be reproduced by such mathematical models. But in integral values of such class, the models can demonstrate high accuracy. This is a specific task of finding appropriate parameters for the developed models.

5. Conclusions

When developing mathematical field models of electrodynamic devices on the basis of Maxwell's equations it is necessary that the systems of coordinates the process is considered in be taken into account. With this factor being ignored, electromagnetic phenomena in the models developed can be misrepresented.

Most of the works dedicated to the analysis of electromagnetic processes in electrodynamic devices on the basis of the theory of electromagnetic field do not focus on coordinate systems, as they apply timeless calculation methods and consider real dynamic of media movement as a series of reciprocal fixed positions of movable and immovable structures. This oversimplifies the physical nature of real electromagnetic processes in devices as they ensure observing the law of total current and not objective regularities (laws) of electromagnetic phenomena in general.

As the results obtained by means of the developed models show, calculations of electromagnetic processes in electrodynamic devices with a constructive periodicity of structures on their pole division should be done by the mathematical field models in the transformed coordinate system. Absence of mechanical movement of media in such models greatly simplifies the model itself, its realization program, and the calculation process.

The method of replacement of complex structure elements by continuous anisotropic media possessing the above mentioned characteristics can be used in mathematical field models of electrostatic and electrodynamic devices. This results in actual changing the device design and in obtaining wrong electromagnetic processes. Only the mathematical field models which most fully take into account design peculiarities of the devices can adequately reproduce electromagnetic phenomena both in the case of dimensional distributions of electromagnetic values and in the case of integral parameters of the devices.

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РОЗРАХУНОК ЕЛЕКТРОМАГНІТНИХ ПРОЦЕСІВ ТУРБОГЕНЕРАТОРА З ЕКВІВАЛЕНТНИМИ ЗУБЦЕВИМИ ЗОНАМИ СТАТОРА І РОТОРА В РЕЖИМІ НЕРОБОЧОГО ХОДУ

Ярослав Ковівчак

На основі рівнянь Максвелла в потенціалах проведено розрахунок електромагнітних процесів у рухомих і нерухомих зонах поперечного перерізу турбогенератора у перетвореній і фізичній системах координат для режиму неробочого ходу. Показано, що використання методу еквівалентування зубцевих зон пристрою суцільними анізотропними середовищами та нехтування систем координат, у яких розглядається процес у польових математичних моделях, приводить до спотворення реальних електромагнітних явищ.

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