Numerical synthesis of the macromodel of generator with soft excitation of oscillations with the help of the second method of Lyapunov

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The summary - **Theoretical substantiation of a method for providing the convergence of a solution of mathematical model of an independent system to a given solution with the help of the second method of Lyapunov is considered. The method is illustrated by model of the self-oscillator, for which the conditions of "soft" excitation of oscillations are provided.**

Keywords - **mathematical simulation, self-oscillator, regularization, Lyapunov method.**

I. THEORETICAL SUBSTANTIATION

Let mathematical model of an autonomous system is given as a system of the differential equations

$$\begin{cases} dy_1/dt = y_2; \\ dy_2/dt = y_3; \\ \vdots \\ dy_n/dt = f(y_1, y_2, \ldots, y_n; \bar{a}); \end{cases} \tag{1}$$

where the nonlinear function $f(Y;\bar{a})$ is continuous, $Y=(y_1, y_2, \ldots, y_n)$, and the vector $\bar{a}$ contains parameters.

It is shown in [1] , that the system (1) models the wide class of systems with concentrated stationary values by parameters.

Some vector function $\tilde{Y}(t) = (\tilde{y}_1(t), \tilde{y}_2(t), \ldots, \tilde{y}_n(t))$; $t \in (t_0, T)$ is given. Then the task of identification of mathematical model (1), which solution is close to a given vector function, looks like:

$$\min_{\bar{a}} \int_{t_0}^{T} \left\| f(\tilde{Y}(t);\bar{a}) - \frac{d\tilde{y}_n(t)}{dt} \right\| dt . \tag{2}$$

If we approximate $f(\tilde{Y};\bar{a})$ in a linear Euclidean space of basis functions $\varphi_i(\tilde{Y}(t))$, $i = \overline{1,m}$, then there exists a unique solution of the identification task

$$\min_{\bar{a}} \int_{t_0}^{T} \left(\sum_{i=1}^{m} a_i \varphi_i(\tilde{Y}(t)) - \frac{d\tilde{y}_n(t)}{dt} \right)^2 dt . \tag{3}$$

In practice the continuous task (3) is substituted by a discrete one owing to digitization of a continuous set (t_0,T):

$$t \in (t_0, T) \Leftrightarrow t_0 \le t_k \le T; \; k = \overline{1,K} .$$

Hence the discrete task of identification is the following:

$$\min_{\bar{a}} \sum_{k=1}^{K} \left(\sum_{i=1}^{m} a_i \varphi_i(\tilde{Y}(t_k)) - \frac{d\tilde{y}_n(t_k)}{dt} \right)^2 . \tag{4}$$

The solution of the discretized task (4) is equivalent to a solution in the quadratic metric of a rectangular system of the linear algebraic equations in relation to a vector of approximating coefficients $\bar{a}$:

$$\sum_{i=1}^{m} a_i \varphi_i(\tilde{Y}(t_k)) = \frac{d\tilde{y}_n(t_k)}{dt}; \; k = \overline{1,K} . \tag{5}$$

The solution (5) always exists and it is unique.

The tasks of identification (2) - (5) are incorrect. It is expressed by the fact that their solutions unacceptably strong depend on errors $\tilde{y}_n(t)$ and from calculation errors. As the result, solution of a system (1) can be somehow far from a given vector $\tilde{Y}(t)$.

Regularization according Tihonov together with a method of reduction of an approximating polynomial [2], [3] (combined regularization) ensures a correctness of the task of identification. However domain of convergence to a solution $\tilde{Y}(t)$ can be too small. Really, the identifications (2) - (5) control the behaviour of the approximated function only along a trajectory $\tilde{Y}(t)$ in a phase space.

Using the ideas of the second method of Lyapunov it is possible to set a desirable domain of convergence of a system solution (1).

Let construct such Lyapunov function in relation to aberrations of a solution $Y(t)$ of a system (1) from a given vector $\tilde{Y}(t)$, that it will be positive in the necessary area, excepting $Y(t) = \tilde{Y}(t)$ points, where it is equal to zero [4].

The choice of the Lyapunov function is very complex task in the case of searching for a stability region of a system [4]. In our case a stability region (that is the domain of convergence to a solution $\tilde{Y}(t)$) is not determined, but it is set. Therefore choice of the Lyapunov function is much easier. A satisfactory sample of such function - incomplete quadratic shape:

$$V(Y) = \sum_{i=1}^{n} \left(y_i(t) - \tilde{y}_i(t) \right)^2 . \tag{6}$$

At such Lyapunov function the domain of convergence can include all phase space.

For $Y(t)$ convergence to $\tilde{Y}(t)$ in some area Ω of a phase space of a system (1) it is enough, that derivative of a Lyapunov function (6) according to equation of motion (1) will be negative in this area, excepting $Y(t) = \tilde{Y}(t)$ points, where it should be zero [4]:

$$\sum_{i=1}^{n} \frac{\partial V(Y)}{\partial y_i} \cdot \frac{dy_i}{dt} < 0 \quad npu \quad Y \in \Omega ; \tag{7}$$

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$$\sum_{i=1}^{n} \frac{\partial V(Y)}{\partial y_i} \cdot \frac{dy_i}{dt} = 0 \quad npu \quad Y = \tilde{Y} . \tag{8}$$

The condition (8) for the function (6) is met always.

Let calculate the left part of an inequality (7) taking into account (6) and (1):

$$\begin{aligned} &\sum_{i=1}^{n-1} \left(y_i(t) - \tilde{y}_i(t) \right) \cdot y_{i+1}(t) + \\ &+ \left(y_n(t) - \tilde{y}_n(t) \right) \cdot f(Y, \bar{a}) \underset{Y \in \Omega}{<} 0; \end{aligned} \tag{9}$$

The condition (9) in discrete variant looks like:

$$\begin{aligned} &\sum_{i=1}^{n-1} \left(y_i(t_k) - \tilde{y}_i(t_k) \right) \cdot y_{i+1}(t_k) + \\ &+ \left(y_n(t_k) - \tilde{y}_n(t_k) \right) \cdot f(Y(t_k), \bar{a}) \underset{Y(t_k) \in \Omega}{<} 0; \end{aligned} \tag{10}$$

From inequalities (10) it is possible to obtain the algebraic equations, which are convenient for supplement of the identification system (5). For this purpose we select such function $\beta(Y)$, that $\beta(Y)<0$ at $Y \epsilon \Omega$ and $\beta(Y) \rightarrow 0$ at $Y \rightarrow \tilde{Y}$. An example is the Lyapunov function (6) with negative sign:

$$\beta(Y) = -\sum_{i=1}^{n} \left(y_i(t) - \tilde{y}_i(t) \right)^2 . \tag{11}$$

Having equated the left parts of inequalities (10) to the function (11), we obtain a system of equations, which is equivalent to the system of inequalities (10) in sense of our task:

$$\begin{aligned} &\sum_{i=1}^{n-1} \left(y_i(t_k) - \tilde{y}_i(t_k) \right) \cdot y_{i+1}(t_k) + \left(y_n(t_k) - \tilde{y}_n(t_k) \right) \cdot \\ &\cdot f(Y(t_k), \bar{a}) = \beta(Y(t_k)); \quad Y(t_k) \in \Omega \end{aligned}$$

Let solve the obtained equations in relation to the approximated function $f(Y(t_k), \bar{a}) = \sum_{i=1}^{m} a_i \varphi_i(Y(t_k))$:

$$\sum_{i=1}^{m} a_i \varphi_i(Y(t_k)) \underset{Y(t_k) \in \Omega}{=} \frac{\beta(Y(t_k)) - \sum_{i=1}^{n-1} \left(y_i(t_k) - \tilde{y}_i(t_k) \right) \cdot y_{i+1}(t_k)}{y_n(t_k) - \tilde{y}_n(t_k)} . \tag{12}$$

The equations (12) have the same structure as equation (5). The mutual solution of the equations (5) and (12) provides the extension of a domain of convergence to a given solution.

It is necessary to select those points $Y(t_k)$ inside area Ω, in which we should control the convergence of a solution $Y(t)$ of the system (1) to a given solution $\tilde{Y}(t)$. The more such points, the more reliably convergence to a given solution in area Ω is guaranteed, but the quality of approximation (5) is declined in this case. It is also necessary to avoid points near a trajectory $\tilde{Y}(t)$ or on it. Then the conditionality of an aggregate algebraic system consisting of the equations (5) and (12) will not be declined.

II. Example of the Method Usage

The inspection of the explained method on a test example of the self-oscillator has shown reliable convergence of a solution of mathematical model to a given solution inside the selected area.

With the use of the method of inverse linear subsystem [1] and combined method of regularization [2], [3] the mathematical model of the self-oscillator as a system of two differential equations with the nonlinear function is constructed. The equations of model are as follows:

$$\begin{cases} dy_1/dt = y_2; \\ dy_2/dt = \omega^2 (f(y_1, y_2) - y_1); \end{cases}$$
$$f(y_1, y_2) = \sum_{i,j=0}^{5} a_{ij} y_1^i y_2^j; \quad i + j \leq 5. \tag{13}$$

In a fig.1 a circuit of the self-oscillator and equivalent circuit of its mathematical model (13) are shown.

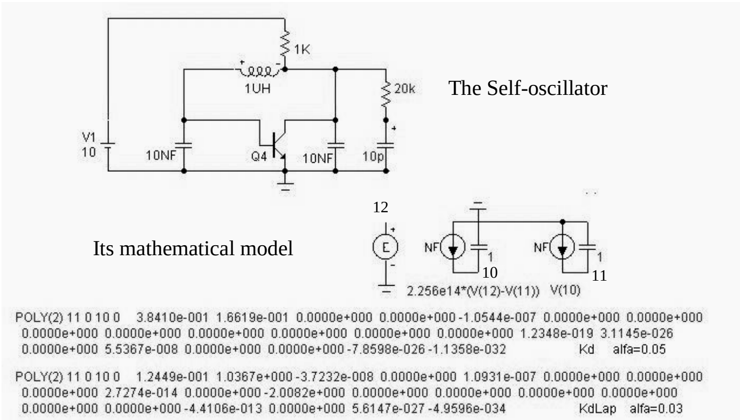

Figure 1. The self-oscillator and its mathematical model described for MicroCap-5.

The differential equations of model (13) are presented in a fig. 1 by two integrating links from linear controlled current sources NF and capacities 1F. State variables y1 and y2 are the voltages of nodes 11 and 10. The nonlinear function $f(y_1,y_2)$ is a fifth degree polynomial of two state variables, this function is presented by controlled voltage source E.

Two variants of the nonlinear polynomial function, shown in a fig. 1, correspond to:

– method of a combined regularization ("Kd");

– method of a combined regularization with the use of the second Lyapunov method ("KdLap").

Output signals of the generator and model are voltage on capacitor 10p and voltage of the 11 node. In fig.2 the output signals of the generator and model with the first variant of the nonlinear function are shown.

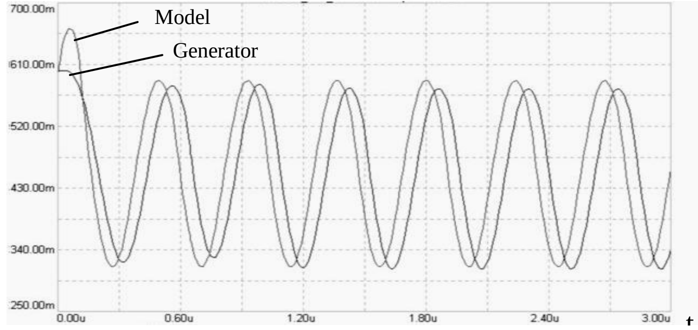

Figure 2. Output signals of the generator and model

The analogs of the equations (5) for model (13), using which the identification of the first variant of the nonlinear function is realized, look like:

$$\sum_{i,j=0}^{5} a_{ij}\tilde{y}^i(t_k)\left(\frac{d\tilde{y}(t_k)}{dt}\right)^j = \frac{d^2\tilde{y}(t_k)}{dt^2}/\omega^2 + \tilde{y}(t_k); ; \qquad (14)$$
$$i+j \le 5;\ \omega^2 = 2.256_{10}14;\ k = \overline{1,400}$$

where $\tilde{y}(t_k)$ is reading of an output signal of the generator in a fig.2 in k-th time moment.

In a fig.3 boundary cycle of oscillations of the self-oscillator on a phase plane in coordinates of output signal and its derivative is shown. The points of the starting conditions of model with the first variant of nonlinearity, at which the integral of model converges to boundary cycle, are presented also.

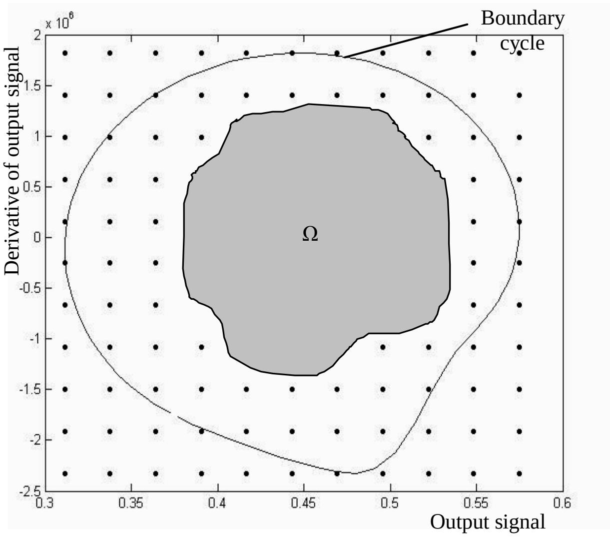

Figure 3. Points – model starting conditions, at which the transient process converges to given boundary cycle. In area Ω the oscillations damp

In a fig.3 it is visible, that for model with the first variant of nonlinearity (obtained by combined method of a regularization) there exists area of the starting conditions, at which the transient process of model damps and does not reconstruct the given output signal of the generator. It corresponds to the so-called "rigid" condition of excitation of oscillations.

Let it is necessary to generate model, which will not have indicated area, that meets the conditions of "soft" excitation. This task we shall solve with the help of a circumscribed above method.

Let construct the Lyapunov function according (6) for boundary cycle in fig.3 with normalized derivative. This function is shown in fig.4.

The analogs of the equations (12) for model (13), which supplement the system (14) for providing the condition of "soft" excitation, are written as:

$$\sum_{\substack{i,j=0\\ i+j\le 5}}^{5} a_{ij}y_1^i(t_k)y_2^j(t_k) = (-\left(y_1(t_k)-\tilde{y}_1(t_k)\right)^2 - \left(y_2(t_k) - \frac{d\tilde{y}_1(t_k)}{dt}\right)^2 - \qquad (15)$$
$$-\left(y_1(t_k)-\tilde{y}_1(t_k)\right)\cdot y_2(t_k)) \Big/ \left(y_2(t_k) - \frac{d\tilde{y}_1(t_k)}{dt}\right); \quad Y(t_k)\in\Omega;$$

where ($\tilde{y}_1(t_k)$, $d\tilde{y}_1(t_k)/dt$) - point of boundary cycle which is the nearest to a point ($y_1(t_k)$, $y_2(t_k)$).

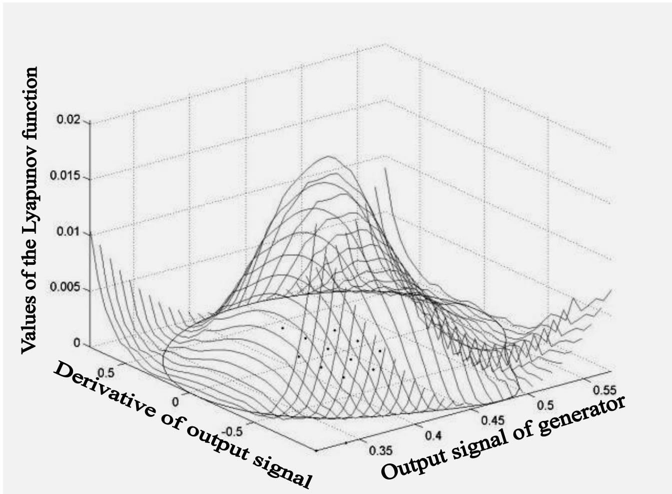

Figure 4. The surface corresponding to Lyapunov function (6). The marked points are the ones, in which additional equations (15) for "soft" excitation of oscillations are composed

In a fig.4 the points of a phase plane are marked, in which the additional equations (15), providing a condition of "soft" excitation of oscillations, are composed according to the explained method.

By mutual solution of 400 basic equations (14) and 11 additional equations (15) we have found the coefficients of the nonlinear function shown in the second variant in fig.1 ("KdLap"). The model with such nonlinear function has no interior area of the starting conditions, for which the periodic regime is not excited, that is corresponds to the generator with "soft" excitation.

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