Parallelization of calculations using GPU in optimization approach for macromodels construction

Abstract. Construction of mathematical models for nonlinear dynamical systems using optimization requires significant computation efforts to solve the optimization task. The most CPU time is required by optimization procedure for goal function calculations, which is repeated many times for different model parameters. The ability to use processor with SIMD architecture or calculation parallelization is the subject of investigation in this paper.

Keywords: macromodels, optimization, parallelization, GPU, SIMD.

Problem Statement
Let's consider some system, which need to be simulated (see fig. 1). Here $\tilde{x}$ - variables which describe system state, $\bar{x}$ - variables which describe external influences, and $\bar{y}$ - variables which describe output values.

$$\begin{align*}
    \tilde{x}(k+1) &= F \cdot \tilde{x}(k) + G \cdot \bar{x}(k) + \Phi(\tilde{x}(k), \bar{y}(k)) \\
    \bar{y}(k+1) &= C \cdot \tilde{x}(k+1) + D \cdot \bar{y}(k)
\end{align*}$$

where $F, G, C, D$ - some matrices with unknown coefficients, which need to be found during model construction, $\Phi(\tilde{x}(k), \bar{y}(k))$ - some vector-function, the form and coefficients of which should also be found.

The mathematical form (1) we selected for our system simulation as well as any other form, has some set of unknown coefficients $\lambda$. In our case it includes elements of matrices $F, G, C, D$ and coefficients of vector-function $\Phi(\tilde{x}(k), \bar{y}(k))$.

Let's introduce some criterion for model precision measuring $Q(\lambda) > 0$, which depicts an inaccuracy of system simulation using the model for known time intervals.

Function $Q(\lambda)$, also called a goal function is generally calculated as a root-mean-square error of system simulation using constructed model.

Therefore model construction can be reduced to finding a value of vector $\lambda$, for which function $Q$ will reach its minimum. This is done using optimization algorithm.

The task of finding a valley point of nonlinear function $Q$ is a complex task, which is considered in many writings. Our goal function is hard to calculate in terms of needed CPU time for one calculation. The time needed for single calculation of goal function depends linearly from the number of samples, because the goal function is a sum of deviations between real values and values simulated using constructed model for all samples. For construction of a high-quality model the number of samples used for goal function calculation will be significant, thus CPU time needed for single calculation of goal function will also be significant.

Practical usage of described method requires a lot of CPU power, so it is worthwhile to consider using parallelization. Taking into account, that goal function calculation for different values of vector $\lambda$ is independent and is done using same algorithm we can state, that processors with SIMD architecture can be effectively used for this task. Such processors allow to execute same instructions for many data sets. SIMD systems includes many identical processor units executing same instruction sets but using independent data sets. Most common device, build on SIMD architecture is GPU (Graphics Processing Unit).

This approach was implemented by authors using CUDA technology. Achieved parallelization effectiveness depends on the number of points for which the goal function is calculated at one step of optimization algorithm. This dependency is shown on fig. 2.

Conclusion
The usage of GPU allows to speed-up the optimization process. One of the simplest GPU cards allowed to speed-up the computation for more than 5 times comparing to relatively new main CPU.

REFERENCES

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