Математична модель метода програмного керування інерціонним об’єктом

Анатолій Гапон[1], Сергій Савицький[2], Наталія Рудакова[3]

Кафедра автоматики та управління в технічних системах, Національний технічний університет “Харківський політехнічний інститут”,61002, УКРАЇНА, м.Харків, вул. Фрунзе, 21,
E-mail: [1.] gapon@kpi.khttpov.ua;
[2.] savitskiy.s@ukr.net;
[3.] rudakova.n@ukr.net

Стаття присвячена розробці математичної моделі екстраполятора системи програмного управління для об'єктів з розподіленими параметрами, які мають властивості лінійного об'єкту з самовирівнюванням. Модель представлена у вигляді рекурентної формули, де враховується перехідні процеси в тепловому об'єкті за допомогою масиву коефіцієнтів відповідності, і яка дозволяє реалізувати процес обчислення реакції теплового об'єкту на базі сучасних мікроконтролерів, що дає можливість розрахувати величину управляючої дії. Модель будується в програмному середовищі Ansys, що дозволяє будувати системи управління з прогнозом для інерційних теплових об'єктів.

Пропонується безперервну перехідну функцію температури теплового об'єкту представити набором дискретних значень в ті ж моменти часу, в які можлива ступінчаста зміна управляючої дії.

Запропоноване рішення доводить ідею прогнозу стану теплового об'єкту до технічної реалізації на базі мікропроцесорного контролера. Стаття представляє інтерес для фахівців в області автоматичних систем, аспірантів і магістрів, що займаються управлінням об'єктами з розподіленими параметрами.

Основні вимоги до системи регулювання, що істотно спрощують синтез системи:

- об'єкт регулювання повинен володіти великою інерційністю, і тому перехід від безперервної системи до дискретної не викличе істотного погіршення характеристик системи;
- об'єкт управління повинен відноситись до класу об'єктів з самовирівнюванням;
- теплофізичні параметри об'єкту управління в заданому діапазоні температур повинні залишатися незмінними;
- вхідний сигнал X(t) - детермінована наперед задана функція часу;
- Рівень перешкод на вході системи, до яких, очевидно, потрібно віднести помилку завдання вхідної величини X(t) і обурюючі дії (коливання температури навколишнього середовища) на систему недбало малі;
- функція X(t) не повинна мати похідних за часом, рівних нескінченності.

Ключові слова – математична модель, системи управління, інерційний об’єкт.

Mathematical model of the inertial object programmed control method

Anatoly Gapon[1], Sergey Savitskiy[2], Natalia Rudakova[3]

Department of automation and control in technical systems, National Technical University "Kharkiv Polytechnic Institute", 61002, UKRAINE, Kharkiv,. Frunze str. 21, E-mail: [1.] gapon@kpi.khttpov.ua; [2.] savitskiy.s@ukr.net; [3] rudakova.n@ukr.net.

The article deals with the development of the extrapolator mathematical model of the software management system for the objects with the distributed parameters, which possess properties of the linear object with self-alignment.

Keywords: mathematical model, management systems, inertial object.

I. Problem definition

The process of control is an organized sequence of logical operations, associated with the transfer of the controlled object from one state into another, designed to compensate the accumulated at the previous stages negative trends and ensure the achievement of eventual outcomes provided the given level of economic effectiveness. Like any other technological process, the control process is not to be implemented immediately, and has certain time dimension, depending on the specific conditions of the control performance, so that the need to consider the loss of time arises. The adaptation process characteristic feature is the current accumulation of data on the system operation process, on the environment and then utilizing it with the aim to improve the chosen quality coefficient. The data accumulation process is connected with the time consumption, which eventually leads to the delay in obtaining the information, necessary for the control system to take a decision. This significantly reduces the efficiency of the control systems performance on a real time basis. Therefore, the problem of predicting the system states (situations), the environment and the system characteristics (behavior) for the adaptive control is still of current interest. Such prediction can be made by utilizing the given method in the control system. The structure, containing the perfect extrapolation link, is often used for the inertial objects. The control theory with predicting considers the methods of the object state extrapolation (predictions) and development of the proactive control action.

Analysis of the controlled object properties, the requirements to the control system and the nature of the system input signal X(t) allows to distinguish the following features, significantly simplifying the control system synthesis:

- the controlled object must have long response time, and therefore the transition from the continuous system to the discrete system will not cause significant deterioration to the system characteristics;

- the controlled object must belong to the class of objects with self-alignment;
- thermophysical parameters of the controlled object must remain unchanged within the required temperature range;
- input signal X (t) – is the deterministic function of time, given in advance;
- the levels of interferences at the input of the system, to which an error of the input variable X (t) setting and disturbing actions on the system (variations of the surrounding media temperature) must be evidently attributed to, are small to negligible;
- the function X (t) should not have the time derivatives, equal to infinite.

This allows making the following simplifications. The continuous variable X (t), which reflects the law of temperature variation in the object, is replaced by a sequence of discrete values X_1, X_2 ,..., H_m. The transient function of the object h (t) is replaced by a set of discrete values K_1, K_2, ..., K_n. The sampling period of the functions X (t) and h (t) is the same and amounts to τ = const. These simplifications allowed synthesizing the block diagram of the thermal control system, which is presented in Figure 2.

The goal objective of the article: to develop the programmed control method with the prediction for the inertial controlled object, provided that the object is inherent the properties of linearity, self-alignment and the superposition principle is valid.

II. Mathematical model of the control method

To implement the proposed method during the preparation period, it is necessary to measure the transient characteristic of the controlled object. The obtained curve allows measuring duration of the transient processes in the object (Fig. 1). By definition, the transient function of the controlled object is a response of the object to the controlling action in the form of the unit function. For the linear objects it is true that the transient function nature does not depend on the magnitude (amplitude) of the control action, and this means that the output object parameter *Y (t)* (temperature increments) to the controlling action X ratio in the form of the unit function is a constant value for the same point in time, for all X values:

$$K = \frac{Y(t)}{X}\Big|_{t=const} = const \qquad (0 < X < X_{max}), \qquad (1)$$

where X_{max} is the maximum value of the control action, which preserves the linear properties of the controlled object. It is obvious that in process of time the *K* coefficient value will be changed until the end of the transient process in the object. In case we divide the object transient function along the time axis into *n* equal intervals τ (Fig. 1 n = 6), we will be able to calculate *Kj* coefficients at the moments of time *t*, multiple of τ, by applying the formula:

$$K_j = \frac{Y(t)}{X}\Big|_{t=t\cdot j} \qquad (1 < j < n), \qquad (2)$$

which univalently determine the transient function of the object at these points for any value of the controlled action:

$$Y(t) = X \cdot K_j\Big|_{t=t\cdot j} \qquad (0 < X < X_{max}; 1 < j < n), \qquad (3)$$

On the other hand, these coefficients *Kj* enable calculating the value of the controlled action *X*, which during the predetermined time $t = j \cdot \tau$ will change the output parameter to the value *Y (η)*.

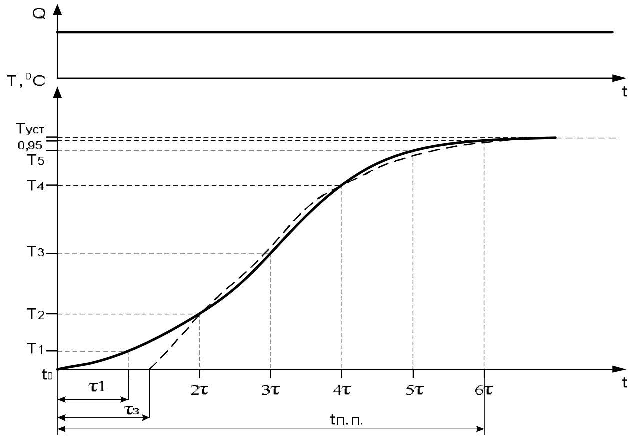

Figure 1. - The transient function of the object under study

On the other hand, these coefficients *Kj* enable calculating the value of the controlled action *X*, which during the predetermined time $t = j \cdot \tau$ will change the output parameter to the value *Y (η)*.

$$X = \frac{Y(h)}{K_j}\Big|_{h=t\cdot j} \qquad (1 < j < n), \qquad (4)$$

Thus, the assignment of the transient characteristic in the form of the coefficient matrix [*K*] allows unambiguous relating of the controlled action value, given in the form of the unit function, with the state of the object under control.

In accordance with the superposition principle, the output variable may be considered as the algebraic sum of the object responses to the elementary controlled actions, the algebraic sum of which fully describes any controlled action. Taking the foregoing into consideration, any stepped function, in the form of which the control action is formed, can be represented as the sum of unit functions:

$$X_i = x_0 + x_1 + x_2 + ... + x_i \qquad (0 < i < \infty), \qquad (5)$$

and the object reaction can be calculated as the sum of responses to the corresponding unit functions:

$$Y_i = y_0 + y_1 + y_2 + ... + y_i \quad (0 < i < \infty), \qquad (6)$$

In Figure 2 there is a block diagram of the system, implementing the proposed method. The given system contains the following component parts: the memory block I of the thermal current increment codes; the memory block 2 of the coefficients *Kj* codes; the computer unit 3 of the predictable temperature change by the end of the control interval under the influence of the heat, supplied by the beginning of this interval; the programmed setting device 4; the comparison element 5 for calculating the code and predictable mismatch error sign; analog - digital converter 6; comparison element 7 for calculating the code and sign of the existing mismatch error; summator unit 8 for calculating the total predictable mismatch error; computer unit 9 for calculating codes of the thermal current increments and the total thermal current. The system also contains the functional generator 10, to calculate the code, which is proportional to the current, which must be supplied to the heater during the next interval of the programmed control; the memory element 11 for storage of the code, proportional to the current into the heater; digital - analog converters 12; the current amplifier 13; the heater 14; the controlled object 15 and the temperature sensor 16.

In the initial state, the memory block 1 and the memory element 11 are set to zero, at that the codes *Kj* are stored into the memory block 2. Into the setting device block 4 there entered the program for changing the temperature of the object at time intervals τ in the form of the temperature increment codes with relation to the initial temperature T_0. To the first input of the comparison element 5 there supplied the temperature increment code, corresponding to the end of the first software control interval $\boldsymbol{\tau}$; to the first input of the comparison element 7 – the zero code correspondingly, i.e. the object temperature increment code at the initial moment of time. To the second input of the comparison element 7 there supplied the code of the object temperature deviation from the object temperature deviation from the T_0 temperature. After starting the software management system the computer unit starts calculating the predictable object temperature changes with relation to the T_0 temperature. To calculate the control action value, which drives the object into the point, specified by the program, during the time interval $\boldsymbol{\tau}$, it is necessary to calculate, to what point will the object be driven under the influence of the controlled action, which was applied prior to starting of the current time interval $\boldsymbol{\tau}$. Therefore, the projected temperature change at the point of time $t=(i+1)\cdot\tau$ must be calculated without taking into account the thermal current, which will be supplied after the time exceeds $t\cdot\tau$ value. Taking this into consideration, this formula will be as follows:

$$\Delta T^{P}_{i+1} = K_n \sum_{j=0}^{i-n+1} \Delta Q_j + \sum_{m=i-n+2}^{i-1} \Delta Q_m \cdot K_{i-m+1}$$

$$i = 1,2,...,\infty \qquad (7)$$

Where ΔT^{P}_{i+1} is the estimated predictable change of the object temperature by the time $t=(i+1)\cdot\tau$ under the influence of the total thermal current, supplied prior to the moment of time $t=i\cdot\tau$; ΔQj - increment of the thermal current controlled action at the moment of time $t=i\cdot\tau$. The formula constitutes an algorithm of the computer unit 3 operation. Graphical interpretation of the predictable temperature change calculating is shown in Figure 3. For display purposes the parameter n is limited to six. The control action (see diagram 1) can be divided into unit functions (diagrams 2, 4, 6, 8). The diagrams 3, 5, 7 and 9 display the object temperature changes under the influence of the corresponding thermal currents.

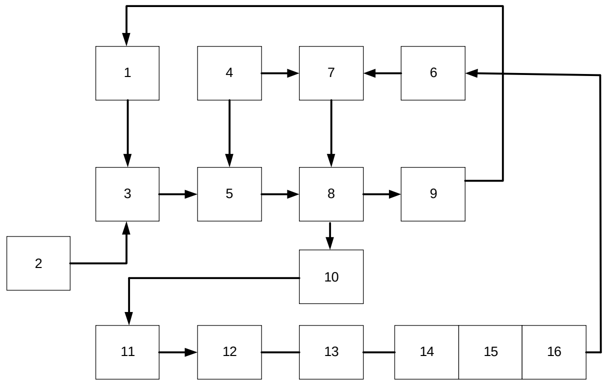

Figure 2 - Block diagram of the object programmed control system

The total change of the object temperature is shown in the Diagram 12 and calculated as the algebraic sum of ordinates of the Diagrams 3, 5, 7 and 9.

Let's calculate the predicted object temperature change at the time t = 10 τ. In accordance with the formula:

$$\Delta T^{P}_{10} = (\Delta Q_0 + \Delta Q_1 + \Delta Q_2 + \Delta Q_3 + \Delta Q_4)K_6 + \Delta Q_5 \cdot K_5 + \Delta Q_6 \cdot K_4 + \Delta Q_7 \cdot K_3 + \Delta Q_8 \cdot K_2 \qquad (8)$$

According to the example, shown in Figure 3, the ΔQ values at the moments of time 3τ, 5τ, 7τ, 8τ and 9τ amount to zero.

In the comparison element 5 the estimated temperature code is compared with the temperature code of the setting device 4. So the difference code with the appropriate sign equals to:

$$\Delta_1 = \Delta T^{3}_{i+1} - \Delta T^{P}_{i+1} \qquad (9)$$

where ΔT^{3}_{i+1} is the temperature increment, required by the program, which comes into the first input of the summator unit 8.

In the course of the control program execution the surrounding media temperature may vary. Since the rate of the ambient medium temperature change is usually significantly lower than that of the object temperature, it is possible to neglect the transient processes, caused by fluctuations in the surrounding media temperature. However, these fluctuations may cause the mismatch error. The round-off errors in coding K_i and ΔQi amount to:

$$d = \frac{A}{2^n} \qquad (10)$$

where A is the maximum value of the encoded value; 2^n the number of memory registers bits, as well as the errors of the digital - analog and analog - digital converters may cause the mismatch error, which must be taken into account. With this aim, to the comparison element 7 there came the setting device temperature increment code for the point of time $t=i\cdot\tau$ and the sensor temperature code, which comes from the output of AD converter 6. The difference code, equal to:

$$\Delta_2 = \Delta T_i^3 - \Delta T_i^g \quad (11)$$

is supplied into the second input of the summator unit 8. Since it is difficult to predict the change in outside temperature, and taking into account that such change over the time $\boldsymbol{\tau}$ is small to negligible, we can accept $\Delta_2(i+1) = \Delta_2(i)$. On this basis we calculate the total deviation of the predictable temperature of the object from the setting device temperature:

$$\Delta = \Delta_1 + \Delta_2 = \left(\Delta T_{i+1}^3 - \Delta T_{i+1}^P\right) + \left(\Delta T_i^3 - \Delta T_i^g\right) \quad (12)$$

Value Δ is calculated by the summator unit 8. It is obvious, that for i = 1, provided that the temperature of the object did not differ from T_0, the value $\Delta T = \Delta T_{i+1}^3$, since all other summands equal to zero.

In order to move the object since the point of time $t=(i+1)\cdot\tau$ into the given point, starting from the time interval $t=i\cdot\tau$, it is necessary to supply the additional thermal current, sufficient enough to cause equal in magnitude but opposite in sign change in temperature.

The thermal current increment value is calculated by the formula:

$$\Delta Q_i = \frac{\Delta}{K_1} \quad (13)$$

where ΔQ_i is the thermal current increment at the moment of time $t=i\cdot\tau$. Since the total temperature deviation Δ is necessary to be compensated for the time equal to $\boldsymbol{\tau}$, the delta Δ should be divided by K_1.

The total thermal current, which is to be supplied within the time interval from $t=i\cdot\tau$ to $t=(i+1)\cdot\tau$, equals to:

$$Q_i = \sum_{j=1}^{i} \Delta Q_j \quad (14)$$

The values of the thermal current increment codes ΔQi and the total thermal current Δ *Qi* are calculated in the computer unit 9, and then the increment code is sent into the memory block 1, and the total thermal current code comes to the input of the functional generator IO. From the output of the functional generator the code, which is proportional to the heater current, is recorded in the memory element 11, and then converted by the DA converter 12 into the analogue signal, which is then amplified by the power amplifier 13 and supplied to the heater 14.

Under the influence of the heat, supplied to the object, it starts to change its temperature to the value ΔT_{i+1}^3. After the code is recorded into the memory element 11, the system starts to compute the control action value for the time interval from $t=(i+1)\cdot\tau$ to $t=(i+2)\cdot\tau$.

Investigation of the object reaction to the controlling actions of the stepped form, the *K* coefficients calculation and the experimental confirmation of the expressions validity were obtained by applying the thermal model in the Ansys medium.

Conclusion

We obtained the expression for predicting the temperature variation of the inertial thermal object when applying the control action of the stepped configuration to it, and then on its basis have constructed the mathematical model and designed the block diagram of the control system with prediction filter.

List of references

[1]. Kolmogorov A.N. The problem of synthesizing the optimal prediction filter. - Izv. of the USSR Academy of Sciences. Series of math. and natur. sciences, № 5, 1941. pages 112-129.

[2.] Weiner N. The Extrapolation Interpolation and Smoothing of Stationary Time-Series. I. Willey, NY, 1949. - 290 p.

[3]. Gabor D., Wilby WR, Woodcock RA A universal nonlinear filter, predictor and simulator which optimizes itself by a learning process. Proc. Inst. Electr. Engrs., Vol. 108., Part B, № 40, 1961. Pp .85-98.

[4]. Ivakhnenko, A.G. and Lapa V.G. Prediction of the random processes.- Kiev, Naukova Dumka, - 1971 - p. 415.

[5]. Sirodzha I.B. Quantum models and methods of artificial intelligence for decision-making and management processes. - Kiev, Naukova Dumka, 2002, - p.490.

[6]. A.G. Butkovskii. Methods of control the systems with distributed parameters. M. Science. Main editorial of the physico-mathematical literature. 1975. - p.568.