Probability Gain of External Signals
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Abstract – In this work a property of macroobjects which consists of randomly cooperating microelements with accidental parameters that stochastically amplify external impacts was discovered. It is shown, that under precritical variance of accidental parameters of microelements increments of probability distribution functions of these parameters exceed corresponding increments of initiating external impacts on macroobject.

Keywords - amplification, probability distribution function, mathematical expectation, dispersion.

I. INTRODUCTION

In statistical radio engineering and the nuclear physics [1, 2], in the quantum theory of crystal firm bodies [3], and also in number of scientific areas connected to them [4], while studying the properties of macroobjects consisting of occasionally reacting with occasional features of microelements, probabilistic theory is usually used for determining quantitative reaction measurements on external effects of macroobject as a whole. At the same time issues on direct connections between external effects on macroobject and variations of probabilistic characteristics of its microelements stay out of view.

Meanwhile, macroobjects that consist of occasionally reacting with occasional features of microelements possess a great feature of stochastic amplification of external effects. While subcritical dispersions of stochastic parameters of microelements increments of probability distribution functions of these parameters exceed increments of external effects on the macroobject. Ascertainment of the fact of existence and practical implementation of stochastic amplification, in our opinion, may change the notion about number of physical phenomena that act as a reaction of macroobjects on such effects identifying quantitative measure of this reaction with reasonable evaluation of probability distribution function of stochastic variables that compose the macroobject of microelements. This also lays the methodological foundation as for detection of effects unknown before, so for creation of fundamentally new means for amplification and control over the processes.

II. MAIN BODY

To prove the existence of stochastic amplification we need to turn to several well known features of probability distribution functions of stochastic variables. Let \( F_X(x) \) be the probability distribution function of stochastic variable \( X \) with zero expectation \( M \) and dispersion \( \sigma^2 \). Obviously probability distribution function of stochastic \( Y = \sigma X + M \) that has \( M \) expectation and \( \sigma^2 \) dispersion will be defined by the next ratio

\[
F_Y(x) = p[\sigma X + M \leq x] = F_X\left(\frac{x - M}{\sigma}\right)
\]

where \( p[\sigma X + M \leq x] \) is probability of that the stochastic variable \( \sigma X + M \) takes value less or equal \( x \).

Suppose that the stochastic variable \( X \) is under external influence. This leads to the fact, that its expectation and dispersion become the time functions: \( M(t), \sigma^2(t) \). Due to this the probability distribution function of stochastic variable \( Y \) will change to:

\[
F_Y(x,t) = p[\sigma(t)X + M(t) \leq x] = F_X\{x(t)\},
\]

where \( x(t) = \frac{x - M(t)}{\sigma(t)} \).

From (2) follows that analysis of changes of probability distribution function of stochastic variable caused by external effects can be led to analysis of distribution function below

\[
F\{x(t)\} = p\{X \leq x(t)\}.
\]

Typical probability distribution function determined by ratio (3) is shown on fig. 1.

Comparison of characteristic shown on fig.1 with well known characteristics of amplifying elements allows us make a conclusion: high enough slope of probability distribution function makes amplification of external effect possible, and it acts like change of probability distribution function of stochastic variable.

As the slope of probability distribution function is defined by

\[
\frac{d}{dx} F\left(\frac{x - M}{\sigma}\right) = \frac{1}{\sigma} w\left(\frac{x - M}{\sigma}\right).
\]

where \( w\left(\frac{x - M}{\sigma}\right) \) is probability density, so the clause of implementation of stochastic amplification will be as shown below [5].
\[ \frac{1}{\sigma} \Phi \left( \frac{x - M}{\sigma} \right) > 1. \] (5)

Taking into account the features of densities of probabilities and \( \delta \) transition, we get
\[ \lim_{\sigma \to 0} \frac{1}{\sigma} \Phi \left( \frac{x - M}{\sigma} \right) = \delta (x - M), \] (6)

where \( \delta (x - M) \) is delta function.

Out of (6) it follows, that while dispersion of stochastic variable seeks to zero, the slope of probability distribution function of this stochastic variable \( e \) is increasing indefinitely in the point that correlates to its expectation.

Let us define critical dispersion of stochastic variable as
\[ \sigma^2 = w^2(0). \] (7)

Meanwhile it is possible to confirm that if the dispersion of stochastic variable is below critical then the amplification of external effect occurs, that is observed as change of probability distribution function of stochastic variable. It means that increment of probability distribution function of stochastic variable will overcome the increment of external effect.

This is proved by the results of stochastic amplification process modeling while reduce of stochastic variable dispersion for equiprobability and Gaussian distribution and quantity implementations equal 1000 [5].

It’s about time to show one of possible approaches to practical implementation of stochastic amplification. Let each \( X_i, \quad i = 1, N \) implementation out of multitude \( N \) implementations of stochastic variable \( X \) is transformed (quantized on the level) due to equation
\[ u_i(t) = \begin{cases} 1 & npu \quad X_i \leq x(t), \\ 0 & npu \quad X_i > x(t), \end{cases} \quad i = 1, N. \] (8)

Then
\[ F^*(t) = \frac{1}{N} \sum_{i=1}^{N} u_i(t) \] (9)
is unbiased and consistent assessment of probability distribution function of stochastic variable \( X \) [6].

Therefore any material object (system, device) that perform sum of transformations (8), (9) while subcritical dispersion of stochastic variable \( X \) always carries stochastic amplification of external effect.

One of the possible variants of stochastic amplification implementation is shown on fig. 2.

![Stochastic amplification implementation scheme](image)

Fig. 2. Stochastic amplification implementation scheme

Amplified signal 1 is given on one of the inputs of comparator 3, occasional signal 2 is given on its other input. Signal 2 can be formed with the help of the noise signal generator or occasional inner noise of comparator 3 can be used. On the output of comparator 3 occasional comparative signal is produced, it shows the probability of immediate values of amplified signal 1 not to outnumber immediate values of signal 2. For example, if immediate values of the amplified signal 1 are lower than immediate values of signal 2, we get high level of signal on the output of comparator 3, otherwise – low level. Thereby, if the dispersion of stochastic variable is less than the critical, signal 1 will be amplified which means that the probability distribution function of stochastic variable has changed. So increment of distributive function of probability of stochastic variable would overcome increments that were cased by signal 1.

The signal, got from output of comparator 3 statistically averages in time (statistically smoothes), that allows to use one of its implementations. Therefore out of signal got from the output of comparator 3 the same signal is being subtracted, delayed on time \( \tau \) in delay element 4 by subtractor 5, and the derived difference is integrated in time by integrator 6. Meanwhile time \( \tau \) is chosen to be equal with time of integration by integrator 6.

### III. CONCLUSION

Equations (8), (9) are satisfied, for example, by characteristics of the process of electron liberation to the zone of conductivity and summarizing charges in cross-section of conductor (that gives an opportunity to touch the problem of electric charge transfer in metal conductors in nonkinetic point of view), photoeffect phenomenon and many other processes in quantum physics. These ratios may become a foundation for creation of supersensitive amplifiers of signals using inherent noises of amplifying elements, and also of photodetectors, discrete antenna arrays. Stochastic amplification naturally underlies analysis techniques on quality of working layers for magnetic and optical data carriers and reduction of damaged data on them. It can be widely used in medicine, for example, in analysis of electrocardiograms, electroencephalograms, organism noises and in other spheres.

### REFERENCES


