PROPERTIES OF POLYPHASE SIGNALS BASED ON THE GENERALIZED FRANK CODES

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Abstract. In this paper the results of different kinds of complex polyphase signals based on Frank codes are given.

Keywords. Complex signals, signal processing, specter, autocorrelation, ambiguity function.

1. Introduction

Effectiveness of the objects and scenes monitoring systems is, to a great extent, defined by the accuracy of the angle and distance coordinates measurements. Such measurements can be realized under any weather conditions, at day or night only by using the radar systems [1]. The characteristics mentioned above, hindrance immunity and reliability of the radar systems greatly depend on a type of the probe signal.

In the radar systems only known signals are formed and radiated, so the parameters of the signals change in the process of their spreading in the space or when they are reflected from different objects [2]. The information, obtaining of which is the task of the radar system, contains in these changed, unknown in advance, parameters of the signals. To determine the properties of such signals, possibilities of their application in the construction of different radio-technical systems it is not enough to know their time and frequency characteristics, but it is also important to know the correlation characteristics of these signals, including the ambiguity functions.

Well known and already applied signals can’t meet the growing demands to monitoring systems. So the search for new algorithms of forming of the new classes of the complex polyphase signals is conducted. One of those signals classes are the signals based on the generalized Frank codes [3].

The properties of the signals mentioned above are not enough researched. In the paper the results of the properties investigations of several different types of the mentioned signals are shown.

2. Theoretical Part

As generalized Frank code (GFC) we define a discrete complex signal which consists of the sequence of elementary signals, amplitude and phase which we get according to the algorithm [4]:

$$\theta_i = \theta_{nm} = nm\frac{2\pi}{N}, 0 \leq n \leq N-1, 0 \leq m \leq M-1;$$

$$A_x = \begin{cases} 1, 0 \leq m \leq L - 1 \\ 0, L \leq m \leq M - 1 \end{cases}$$

where $N$ – signal phase levels value.

Generalized Frank code is formed as follows. A time domain equal to the duration of the signal $T_3$ is divided in $S=MN$ temporal positions of duration $T_0=T_3/S$. On each of these temporal positions radio signals with frequency $\omega_0$ are formed, rounding and initial phase which is chosen according to the algorithm (1).

The algorithm of GFC forming becomes clearer when the following complex matrix, shown in figure1, is used.

A radio frequency signal is created by the row-to-row writing down of the matrix and transferring of the sequence on the carrier frequency

$$\chi(r, f) = \sqrt{A_i^2 + B_i^2 + 2A_iB_i\cos\left(\left(NL - 2L + 1\right)\frac{\pi}{N}\right)}$$

where

$$A_i = \frac{\sin\left((L-g)\frac{\pi}{N} - \frac{\pi f}{Af}\right)}{\sin\left((L-g)\frac{\pi}{N} - \frac{L\pi f}{Af}\right)} \times \frac{\sin\left((N-r)\frac{\pi f}{Af}\right)}{\sin\left((r+1)\frac{\pi f}{Af}\right)}$$

$$B_i = \frac{\sin\left((N-r-1)\left(L-g\frac{\pi f}{Af}\right) - \frac{L\pi f}{Af}\right)}{\sin\left((L-g\frac{\pi f}{Af}) - \frac{L\pi f}{Af}\right)} \times \frac{\sin\left((g(r+1)\frac{\pi f}{Af})\right)}{\sin\left((r+1)\frac{\pi f}{Af}\right)}$$

$k = rN + g; 0 \leq r \leq N - 1; 0 \leq g \leq N - 1$

$f$ – Doppler frequency shift.
where

$p = L/N = 1$

GFC transforms into a Franck code. By substituting

which have only the dependencies on an amplitude and

complex and hard for calculations by using computer, it

was decided to form the signals as analytical signals,

Because the given equations  (2) and (3) are rather

complex and hard for calculations by using computer, it

was decided to form the signals as analytical signals,

which have only the dependencies on an amplitude and

phase multiplier, and the carrier frequency is supposed to

be known.

The investigation of the complex polyphase signals

was conducted by using a specially created computer

program within MATLAB system. When

$L = M = N$ the GFC transforms into a Franck code. By substituting

$p = L/N = 1$ into (2) and (3) we receive the equations for

Frank code ambiguity function.

For $M \geq 2L$

\begin{equation}
\chi(\tau, f) = \left( A_1, \text{for } 0 \leq g \leq L - 1; \right.
\begin{array}{ll}
0, & \text{for } L - 1 \leq g < M - L + 1; \\
(B_2), & \text{for } M - L + 1 \leq g \leq M - 1. \\
\end{array}
\right)
\end{equation}

where

\begin{equation}
A_1 = \sin \left( \frac{r}{N} \left( r - \frac{\pi f}{N} \right) \right) \times \sin \left( \frac{N - r}{N} \left( \frac{g}{N} - \frac{\pi f}{N} \right) \right)
\end{equation}

\begin{equation}
B_2 = \sin \left( \frac{(M - L - g)}{N} (r + 1) \frac{\pi f}{N} \right) \times
\sin \left( \frac{(N - r - 1) (M - g)}{N} \frac{\pi f}{N} \right) \times
\sin \left( \frac{(M - g) \pi f}{N} \right)
\end{equation}

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$L = M = N$ the GFC transforms into a Franck code. By substituting

$p = L/N = 1$ into (2) and (3) we receive the equations for

Frank code ambiguity function.

For $M = L$

\begin{equation}
\chi(\tau, f) = \sqrt{A_1^2 + B_2^2 + 2A_1B_2 \cos \left( \frac{(N^2 - 2N + 1) \pi}{N} \right)}
\end{equation}

where

\begin{equation}
A_1 = \frac{\sin \left( \frac{(N - g)}{N} \left( \frac{r \pi f}{N} \right) \right) \sin \left( \frac{(N - r)}{N} \left( \frac{g \pi f}{N} \right) \right)}{\sin \left( \frac{- \frac{r \pi f}{N}}{N} \right) \sin \left( \frac{r + 1}{N} \frac{\pi f}{N} \right)}
\end{equation}

Below, the results of some characteristics of the

GFC are given.

3. Results

For the GFC with the value of phase quantization

$N=4$ and $p=1$ Fig.1 shows the autocorrelation function

(ACF), the main lobe of which is rather wide and there is

a significant level of side lobes. With the increase in the

phase quantization value to 7 with $p=1$ the width of the main lobe of the ACF decreases and the level of the side

lobes becomes lower (fig. 2).

The specter of the signal based on the GFC for

$N=12$ and $p=0.5$, its ACF, frequency ACF and

ambiguity function are shown on the fig. 3-6.

Analogically for the signal based on the GFC with

$N=12$ and $p=2$ the corresponding characteristics are

shown on the fig. 7-10, and on the fig. 11-14 for the signal based on the GFC with $N=12$ and $p=2$.

As the Figures show, the character of the specters of

the signals based on the GFC for all $N$ and $p$ parameters

is saved, but for $p<1$ and $p>1$ the specter will have the

striped structure. The width of the stripes and their

number in the lobes depends on the parameter $p$ and

phase quantization level $N$. Also the main lobe of the

ACF is split, i.e. two side lobes with big amplitude

appear.
Properties of Polyphase Signals Based on the Generalized Frank Codes

Fig. 1. Autocorrelation function of the signal based on the GFC with parameters $N=4$, $p=1$

Fig. 2. Autocorrelation function of the signal based on the GFC with parameters $N=7$, $p=1$

Fig. 3. Specter of the signal based on the GFC with parameters $N=12$, $p=0.5$

Fig. 4. Autocorrelation function of the signal based on the GFC with parameters $N=12$, $p=0.5$

Fig. 5. Frequency-autocorrelation function of the signal based on the GFC with parameters $N=12$, $p=0.5$

Fig. 6. Ambiguity diagram of the signal based on the GFC with parameters $N=12$, $p=0.5$

Fig. 7. Specter of the signal based on the GFC with parameters $N=12$, $p=1$

Fig. 8. Autocorrelation function of the signal based on the GFC with parameters $N=12$, $p=1$
Fig. 9. Frequency-autocorrelation function of the signal based on the GFC with parameters $N=12$, $p=1$

Fig. 10. Ambiguity function of the signal based on the GFC with parameters $N=12$, $p=1$

Fig. 11. Specter of the signal based on the GFC with parameters $N=12$, $p=2$

Fig. 12. Autocorrelation function of the signal based on the GFC with parameters $N=12$, $p=2$

Besides, the ACF of the signal based on the GFC with $N=11$ and $p=1$ is shown in Fig. 15, and in Fig. 16 – for the signal with $N=16$ and $p=1$.

Fig. 13. Frequency-autocorrelation function of the signal based on the GFC with parameters $N=12$, $p=2$

Fig. 14. Ambiguity function of the signal based on the GFC with parameters $N=12$, $p=2$

Fig. 15. Autocorrelation function of the signal based on the GFC with parameters $N=11$, $p=1$

Fig. 16. Autocorrelation function of the signal based on the GFC with parameters $N=16$, $p=1$

For the signal with $p<1$ levels of the ambiguity pikes in the field $(fT) \leq N/2$ are defined by the equation
Properties of Polyphase Signals Based on the Generalized Frank Codes

\[ U' = \frac{2}{c \pi} \frac{\sin(c + \eta / p)}{\sin(\eta)}, \quad c \in N \] (5)

where \( \eta = \frac{fT}{N} \), \( c \) – ordinal number of the ambiguity spike.

For the signals with \( p > 1 \), the specter stripe’s fronts are not so steep as the signals with \( p < 1 \) have. In these signals the main lobe of the ACF is split, i.e. two side lobes with a big amplitude appear. It is necessary to note that the specter striped structure and the split of the ACF main lobe does not have a great influence on the ambiguity diagram in general, as in the case of the signals based on the GFC with \( p < 1 \).

Levels of the ambiguity pikes in the field \( (fT) \leq N / 2 \) are defined by the equation

\[ U' = \left( \frac{\sin(\pi \left( 1 - \frac{c}{p} - \eta / p \right))}{\pi \eta} \right) \cdot 0 \leq \eta \leq 1 \] (6)

where \( c = 1, 3, \ldots \) – ordinal number of the ambiguity spike.

The important property of the signals based on the GFC is the side lobes decrease in the comparison with the signals, which are used in modern radars. So the numerical results of the side lobes levels for different signals are shown in Table 2, and their levels dependency on the phase quantization levels \( N \) for GFC is shown in Fig. 17. As we can see, with the growth of the \( N \) the level of the side lobes decreases.

**4. Conclusions**

The signals based on the GFC with the corresponding phase quantization levels give us an opportunity to form needle-shaped autocorrelation functions with low levels of the side lobes on the time-Doppler frequency field. Their use in the radar systems will lead to the growth of the system’s resolution, hindrances immunity and reliability of the detection and recognition.

**References**


![Fig. 17. Side lobes dependency of the signals based on the GFC with different parameters of N and p=1](image_url)

**Table 2**

<table>
<thead>
<tr>
<th>Signal</th>
<th>Side lobe level, max</th>
</tr>
</thead>
<tbody>
<tr>
<td>LFM</td>
<td>0.28</td>
</tr>
<tr>
<td>Barker code, 13-positional</td>
<td>0.1</td>
</tr>
<tr>
<td>Biphasic recurrent code, 127-positional</td>
<td>0.07</td>
</tr>
<tr>
<td>5-phase recurrent code, 124-positional</td>
<td>0.08 (0.05 mean)</td>
</tr>
<tr>
<td>Franck code, 11-phase levels, 121-positional</td>
<td>0.029</td>
</tr>
</tbody>
</table>
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